

# Displacement Estimation by Savitzky Golay Filtering Technique through Inertial Sensor

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Contactless displacement measurement using low-cost sensor modules is vital in engineering applications like elevators. To achieve contactless displacement measurement with high resolution and reliability, a new method of Displacement Estimation on Navigators (DEON) is proposed. The purpose of the proposed approach is to estimate the navigator's motion using a micro electro-mechanical system based accelero-meter and to compute the actual displacement covered by the navigators. The error profiles are analysed in acceleration signal which is captured by the accelero-meter and it is extracted and calibrated to remove the offset drift error. Further, the data has been processed to reduce random drift error by applying the Savitzky-Golay filter. Then, the principles of integral calculus are applied in the time-domain and displacement is obtained by integrating acceleration twice. The proposed algorithm aimed at measuring displacement caused by the vibration to the object. The similarity between actual and estimated displacement is 98%.

**Keywords:** DEON, Error Profiles, Drift error, Savitzky-Golay filter, Accelerometer.

## 1 Introduction

The rapid development of microelectronics technology leads to the pavement for the miniaturization of devices. Manufacturing Micro Sensors by Micro electro mechanical system (MEMS) technology meet the high-speed performance requirements and it massively reduces the cost of production. This maturity of MEMS replaces traditional mechanical sensors and being penetrated high-end applications [1-4]. To measure the motion characteristics of a navigator in engineering applications, MEMS accelerometers are used. In the existing methods [5,6], ultrasonic waves and lasers are used because of its higher accuracy in computing displacement, but the measurement depends on the feedback of objects. Therefore, accurate contactless displacement measurement of the motile objects is highly in need. The measured coordinates from the accelerometer are usually contaminated by unrealistic errors (drifts) and these errors are serious in long displacement computation. This paper presents a novel technique of analyzing and processing sensor's error. The least squares principle is used to obtain error profiles to correct dynamic offset noise and the drift errors are rectified sufficiently by using Savitzky-Golay filter[7-9] which provides a viable solution to integration methods to reduce the accumulation of trend term error [10,11]. The integration is done in the time domain using the trapezoidal rule and thereby DEON computes displacement with a higher accuracy.

## 2 Methodology

In this paper, the 3 axis accelerometer is used as a data acquisition device and can measure up to 16g. This work focuses on one-dimensional computation and so the acceleration for the x-axis alone is taken for measurement. The navigator which is used for the study is remote controlled vehicle. The data collected by the accelerometer is usually measured in  $m/s^2$  is corrupted by noise. Therefore, sensor measurement noise will be accumulated in the computing process and thus affects the accuracy of the measured displacement result. To estimate the actual displacement from the measured acceleration, errors have to be analysed. The proposed DEON method analyses error deeply and sufficiently deducted from the acceleration signal. The workflow of DEON is shown in fig 1. The entire work flow of DEON approach is described in the following sections.

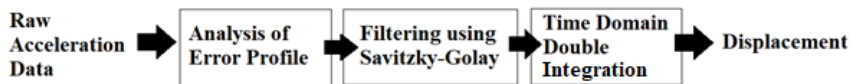


Fig.1. Workflow of DEON approach

### 2.1 Extraction of Error Signatures

Under practical conditions, the acceleration signal is interfered with noise such as random drift errors and dynamic offset error. Mathematically the measured acceleration data  $a(t)$  can be expressed as

$$a(t) = f(t) + E \quad (1)$$

where,  $f(t)$  represents the actual acceleration and  $E$  represents drift error term.

To eliminate the presence of offset and drift errors and if there is a minimum sum of squares of differences between polynomial function  $a_m(t)$  and data points  $(t_i, a_i)$  ( $i=0,1,2,3...n-1$ ) then by using least square method [12,13] to extract error profile of acceleration data is expressed as,

$$a_m(t) = \sum_{k=0}^h c_k t^k \quad (2)$$

where,  $h$  is the highest order coefficient,  $c_k$  is the coefficient of  $a_m(t)$  and  $n$  is the total sampled data points.

$c_k$  is obtained such that  $a_m(t)$  takes the minimum value and thereby integral trend term can be determined. The acceleration calculation formula is

$$a'_i = a_i - (c_1 t + c_0) \tag{3}$$

where,  $a'_i$  is the corrected acceleration data and  $a_i$  is the acquired acceleration data. To attain real-time computation, the time  $t$  can be written as

$$t = i/f_s, \quad i = 0, 1, 2, \dots, n - 1 \tag{4}$$

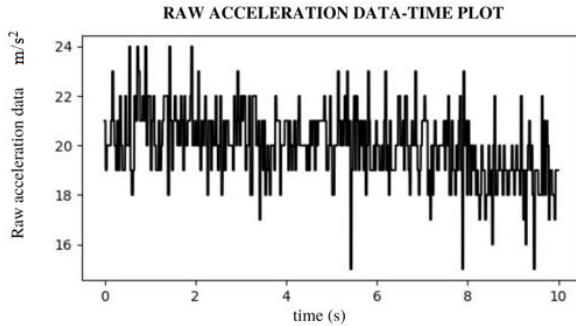
where the Sampling frequency  $f_s$  of the sensor is fixed. The solution of linear equation is obtained as,

$$c_0 = \frac{[D_i \sum_{i=0}^{n-1} a_i - C_i^2 \sum_{i=0}^{n-1} (B_i a_i)]}{[n D_i - C_i^2]}, \quad c_1 = \frac{[n \sum_{i=0}^{n-1} (B_i a_i) - C_i \sum_{i=0}^{n-1} a_i]}{[n D_i - C_i^2]} \tag{5}$$

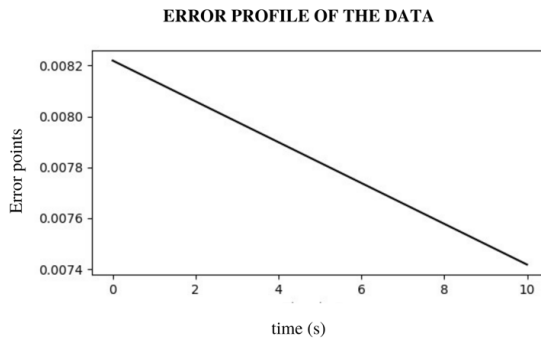
where,  $B_i = (i/f_s), C_i = \sum_{i=0}^{n-1} (i/f_s), D_i = \sum_{i=0}^{n-1} (i/f_s)^2$

Substitute equation 5 in 3 to get accurate raw acceleration data.

It is clear from the fig.2 that, in the stationary period, the acceleration data is hugely corrupted by drift errors. After applying the least square method, the error profile of acceleration is obtained as shown in fig.3 and the error signatures of every time instant obtained from this error profile will be deducted from raw acceleration results in zero offset error.



**Fig. 2.** Raw acceleration data of a navigator for period 10s under stationary state



**Fig. 3.** Error profile plot of a navigator for period 10s under stationary state

**2.2. Filtering Using Savitzky-Golay**

Though dynamic offset error is reduced by the least-squares method, there are some random drift errors still in the acceleration signal. This can be smoothed by the Savitzky-Golay filter which also utilizes the least square principle. The Savitzky-Golay (SG) filter adopts a polynomial in a sliding window to fit the acceleration signal piecewise depends on the least-squares algorithm and it is being used as a tool for denoising data. It is smoothens data without changing the original data properties [14].

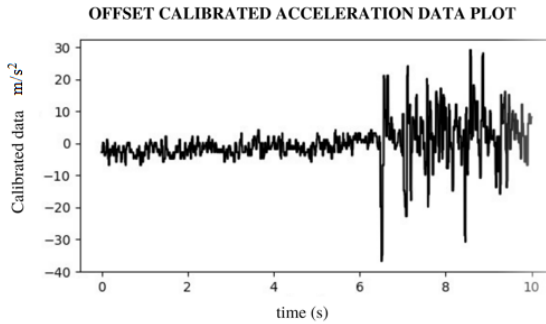
Mathematically, the SG filter can be expressed as

$$a_{SGj} = \frac{\sum_{i=-m}^{i=m} s_i a'_{n+1}}{2m+1} \tag{6}$$

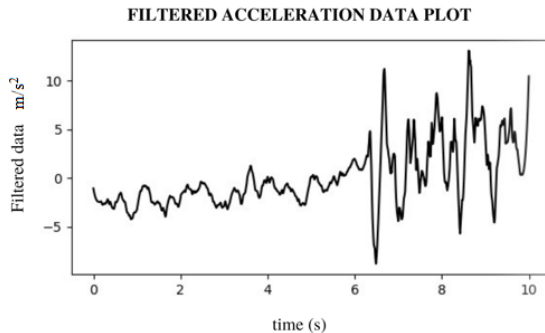
where,  $a_{SG}$  is the smoothed signal,  $s_i$  are the  $i^{th}$  smoothing coefficient and  $m$  is the half-width of the smoothing window and  $j$  represents the running index of  $a'$ .

Fig 4, 5 shows the offset calibrated raw acceleration, filtered raw acceleration data for a period of 10s under the motion of 2.3 cm respectively. From the fig.5, it is clear that, the Savitzky-Golay filter sufficiently smoothen the data with the window size of 51 data points.

According to the principle of calculus, the displacement is obtained by integrating acceleration twice. Integration of raw signal results in large trend term errors. But in the DEON method, the raw acceleration signal is highly processed by the least-squares method and SG filtering technique. So, integrating the processed signal will not end in large errors and the estimated displacement results have been verified.



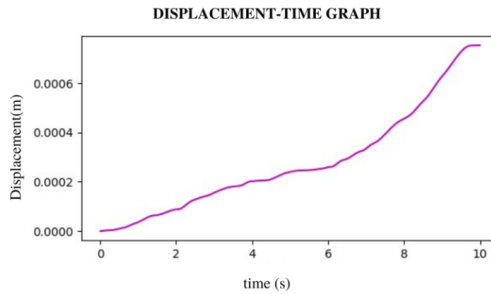
**Fig. 4.** Offset calibrated raw acceleration of a navigator in the motion of period 10s



**Fig. 5.** Filtered acceleration data of a navigator in the motion of period 10s

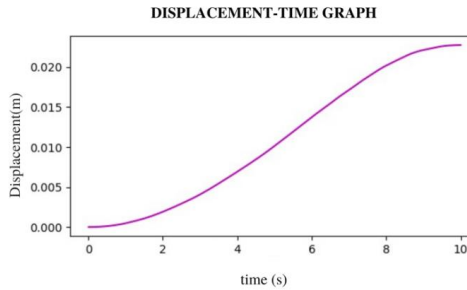
### 3 Results and Discussion

The proposed DEON approach analyses error and eliminating those errors by using the least-squares method and Savitzky-Golay filtering technique to achieve accurate displacement results. The paper also displays the acceleration signal plot in both stationary and motion periods of a navigator. Figure 6 show the displacement measured in stationary time and it is clear that under navigator in stationary period, the displacement measured is (0.00065m) which is negligible in consideration.



**Fig.6** Displacement-time plot captured by a navigator for time 10s under the stationary state

Figure 7 show the displacement measured for 2.3cm for the time of 10s and the estimated displacement is 2.25cm which is significant.



**Fig.7** Displacement-time plot captured when the navigator is in motion for 10 s.

Table 1 shows the validation of closeness of measured different motion lengths to the actual displacement lengths. The displacements in the table 1 have been measured for the period of 10s.

**Table 1.** Similarity between measured and actual displacement lengths

S. No.	Actual Displacement (cm)	Measured Displacement (cm)	Portion of error (%)	The closeness between actual and measured displacement (%)
1.	1	1.0206	2.060	97.940
2.	5	5.104	2.080	97.920
3.	25	25.525	2.100	97.900
4.	50	55.60	1.120	98.880
5.	100	101.11	1.110	98.890
6.	200	203.04	1.152	98.848
7.	250	253.015	1.206	98.795
Average:			2(approx)	98(approx)

It is inferred that on average, the DEON approach exhibits a high correlation of 98% to the actual displacement and the portion of error is minimal. Therefore, the DEON approach exhibits negligible variations in measured results compared to the actual displacement.

## 4 Conclusion

The displacement has been estimated by the use of high SNR filtering technique imposed on acceleration data. This work has been tested with very small distance measurement and clearly exhibits the high correlation between actual and estimated measurements. This work finds an application where the small displacement is important to note and also this is clear that computing large displacement will not end in irreparable drift errors. Also the algorithmic study has been done with one dimensional computation and this approach can also be extended to 2D or 3D positioning systems. In the future, an indoor positioning system will be developed based on DEON approach to find the GPS coordinates when there are GPS outage issues.

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