Newton's Method in Matlab. 19.0 to Minimize a Function of Direct Solar Infrared Radiation (2100nm to 4000nm)

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This research work has as main objective to apply Newton's method in Matlab. 19.0 to minimize a nonlinear spread function infrared direct solar radiation (2100 nm to 4000 nm). The methodology that was applied consisted of the analysis of the components of direct infrared radiation: Rayleigh scattering, aerosols, water vapor between the atmosphere and the Earth, to generate a theoretical model of infrared radiation that depends on of the parameters: turidity coefficient β and length of the ozone layer l, precipitable water vapor W. In the which this model was compared with experimental data through of a nonlinear Dispersion Function, that was minimized through Newton's method, obtaining as a result the theoretical model of infrared radiation using the parameters: β = 0.9499, l = 1.0889 cm and predominant in W= 3.8342 cm. for the Costa Rica station, corresponding to a cloudy white sky atmosphere without the presence of an ozone hole with a considerable presence of water vapor.

Keywords: Minimization, infrared, water vapor, turidity coefficient.

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1 Introduction

Newton's method plays a central role in the development of numerical techniques for optimization with numerous applications in computer science, renewable energy, and management, data mining, industrial and financial research [2]. It is therefore; that is one of the reasons of its importance and that arises quite naturally when considering a convex function. Due to its simplicity and wide applicability, Newton's method remains an important tool for solving many optimization problems [3]. Therefore, Newton's method is used to solve equations with non-differentiable function with sufficient conditions of convergence and estimates of both the speed and the range of convergence [4]. Also there was interest in applying second-order methods, such as Newton-based methods that build Hessian approximations using only gradient in-formation to study the behavior of Newtonian stochastic algorithms to train deep neural networks [5].

On the other hand, infrared radiation is very important in the development of energy advances through efficient solar technologies for the production of photovoltaic, thermoelectric and thermal energy as a sustainable source with a greater future due to its low cost or energy storage capacity [6]. The effects of attenuation by absorption and scattering by infrared solar radiation on a horizontal surface were proposed by Berlage [7]. Leckner proposed a simple formula for solar infrared radiation which took into account aerosol scattering, absorption by ozone, water vapor, and uniform-ly mixed gases. Therefore; it is necessary to minimize convex functions of infrared solar radiation based on Newton's method to determine atmospheric models with experimental meteorological data of infrared radiation.

2 Methods:

2.1 Theoretical model of infrared solar radiation (2100 nm a 4000 nm)

The theoretical model of the direct spectral solar radiation (290 nm to 4000 nm) given in equation 1 is the product of the extraterrestrial radiation with the different transmittance expressions and by the Earth-Sun distance correction factor, without having consider the diffuse radiation coming from the whole sky and is given by [8]:

$$I_\lambda(\lambda, w, l, \beta) = I_{o\lambda} E_o e^{-\frac{m_a}{\left[\lambda^4\left(115.6406-\frac{1.335}{\lambda^2}\right)\right]}-\frac{0.2385 k_{w\lambda} w m_r}{\left(1+20.07 k_{w\lambda} w m_r\right)^{0.45}}-\frac{1.41 k_{g\lambda} m_a}{\left(1+118.93 k_{g\lambda} m_a\right)^{0.45}}-k_{o\lambda} l m_o-\beta k_{a\lambda} m_a} \tag{1}$$

Where:
λ: Wavelength (290 nm a 4000 nm).
β: Aerosol turbidity coefficient (dimensionless).
l: Ozone layer length (cm).
w: Water vapor length (cm).
$I_{o\lambda}$: Extraterrestrial radiation intensity ($Wm^{-2}\mu m^{-1}$).
$I_\lambda(\lambda, w, l, \beta)$: Theoretical direct radiation intensity ($Wm^{-2}\mu m^{-1}$).
m_r: Relative air mass [9].
m_o: Relative mass of ozone (dimensionless) [10].
m_a: Relative aerosol mass (dimensionless) [11].
E_o: Correction factor for the Sun-Earth distance (dimensionless) [12].
$k_{o\lambda}$: Ozone attenuation coefficient (dimensionless).
$k_{a\lambda}$: Aerosol Attenuation Coefficient (cm^{-1}).
$k_{g\lambda}$: Attenuation coefficient by gas absorption (dimensionless).
$k_{w\lambda}$: Water vapor attenuation coefficient (cm^{-1}).

2.2 Selection of experimental data

From the experimental spectrum of the Costa Rica station in Figure 1 [1], data were extracted for 20 wavelengths of direct infrared solar radiation for latitude 10° 02'N, longitude 84° 09'W, altitude of 1050 m.sn.m) on August 20, 2002, using the Excel and Paint programs with a resolution of 100 nm of the solar spectrum shown in Table 1 for (2100 nm to 4000 nm).

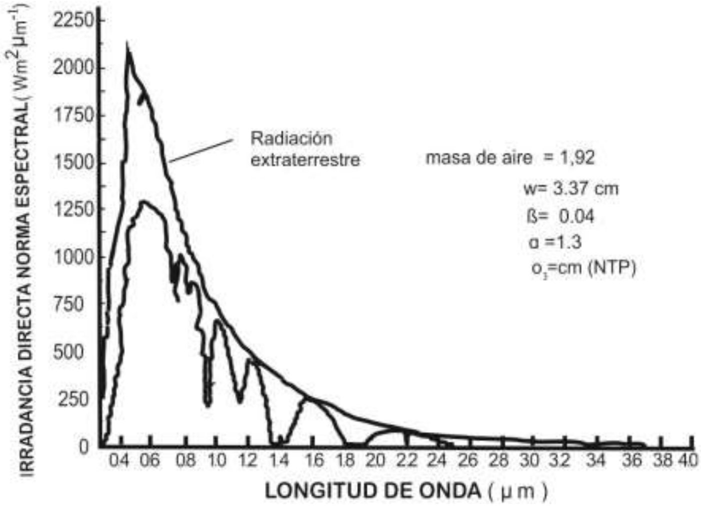


Figure 1. Direct irradiation, normal, spectral radiation under specific atmospheric transmission conditions [1].

Table 1. Solar infrared irradiance (2100 nm to 4000 nm), extraterrestrial $I_{o\lambda}$ WRC (World Radiation Center) and experimental direct [1] and aerosol absorption coefficients $k_{a\lambda}$ [13], ozone $k_{o\lambda}$ [14], water vapor $k_{w\lambda}$ [8] and uniformly mixed gases $k_{g\lambda}$ [8].

λ (μm)	$I_{o\lambda}$ (Wm^{-2}μm^{-1})	$I_{n\lambda}(\lambda, W, l, \beta)$ (Wm^{-2}μm^{-1})	$k_{o\lambda}$ (cm)$^{-1}$	$k_{a\lambda}$ (μm)$^{-1}$	$k_{g\lambda}$	$k_{w\lambda}$
2,100	96,89	86,96	0	0,3812	0,240E+00	0,220E+00
2,200	64,59	32,61	0	0,3588	0,380E-03	0,330E+00
2,300	53,83	54,35	0	0,3387	0,110E-02	0,590E+00
2,400	53,83	21,74	0	0,3204	0,170E-03	0,230E+02
2,500	53,83	0,00	0	0,3039	0,140E-03	0,310E+03
2,600	43,06	0,00	0	0,2888	0,660E-03	0,150E+05
2,700	32,30	0,00	0	0,2749	0,100E+03	0,220E+05
2,800	32,30	0,00	0	0,2622	0,150E+03	0,800E+04
2,900	21,53	0,00	0	0,2505	0,130E+00	0,650E+03
3,000	21,53	0,00	0	0,2397	0,950E-02	0,240E+03
3,100	21,53	0,00	0	0,2297	0,100E-02	0,230E+03
3,200	10,77	0,00	0	0,2204	0,800E+00	0,100E+03
3,300	10,77	0,00	0	0,2118	0,190E+01	0,120E+03
3,400	10,77	0,00	0	0,2037	0,130E+01	0,195E+02
3,500	10,77	0,00	0	0,1962	0,750E-01	0,360E+01
3,600	10,77	0,00	0	0,1892	0,100E-01	0,310E+01
3,700	10,77	0,00	0	0,1825	0,195E-02	0,250E+01
3,800	10,77	0,00	0	0,1763	0,400E-02	0,140E+01
3,900	10,77	0,00	0	0,1705	0,290E+00	0,170E+00
4,000	10,77	0,00	0	0,1649	0,250E-01	0,450E-02

2.3 Convex nonlinear scattering function of direct infrared solar radiation (2100 nm a 4000 nm)

To validate the theoretical direct infrared radiation, it was compared with the experimental direct infrared radiation $I_{n\lambda}$ for the Costa Rica station using a nonlinear dispersion function

$$f(\lambda, W, l, \beta) = \sum_{\lambda} \left[\frac{I_{n\lambda}(\lambda) - I_{\lambda}(\lambda, W, l, \beta)}{I_{n\lambda}(\lambda)}\right]^2 \qquad (2)$$

$$0 \leq \beta \leq 1{,}0$$
$$0 \leq l \leq 1.2$$
$$0 \leq W \leq 5.0.$$

To minimize the function f, the Kun-Tuker conditions [15] and the internal penalty technique [16] are applied, transforming it into a function of equality restrictions, obtaining a matrix function F_{μ} given by:

$$F_{\mu}(w, \lambda) = \begin{bmatrix} \nabla f(w_1 + L_1, w_2 + L_2, w_3 + L_3) - \begin{bmatrix} \frac{\mu}{w_1} + \lambda_1 \\ \frac{\mu}{w_2} + \lambda_2 \\ \frac{\mu}{w_3} + +\lambda_3 \end{bmatrix} \\ -\frac{\mu}{w_4} + \lambda_4 \\ -\frac{\mu}{w_5} + \lambda_5 \\ -\frac{\mu}{w_6} + \lambda_6 \\ w_1 + w_4 + L_1 - U_1 \\ w_2 + w_5 + L_2 - U_2 \\ w_3 + w_6 + L_3 - U_3 \end{bmatrix}. \qquad (3)$$

Where:
μ: Penalty constant.
λ_i: Lagrangian constants ($i = 1{,}2$)
w_i: Slack variables ($i = 1{,}2$)
L_i, U_i: Boundary values for β y l ($i = 1{,}2$).
Making

$$\xi = \begin{bmatrix} W \\ \lambda \end{bmatrix}, \qquad (4)$$

the function F reduces to:

$$F_{\mu}(\xi) = 0, \qquad (5)$$

taking care that

$$\xi_1 = w_1, \ldots, \xi_6 = w_6 > 0, \qquad (6)$$

and his Jacobian

$$\left[J_{\mu}(\xi)\right] = \frac{F_{\mu}(\xi + he_i) - F_{\mu}(\xi)}{h}, i = 1, \ldots, 6. \qquad (7)$$

Where h is a small value.

2.4 Newton's method

The function f was minimized by Newton's method using the following basic implementation [17].
function Newton (x, y, λ)
while $|F(x, y, \lambda) > \varepsilon|$
Solve $J(x, y, \lambda) = \begin{bmatrix} dx \\ dy \end{bmatrix} = -F(x, y, \lambda)$ (get dx y dy)
Find $\alpha \leftarrow \min_i = 1 \ldots n\{-\frac{dx_i}{dx_i} : dx_i < 0\}$ (criterion of reason)
Do $x \leftarrow x + 0.98\alpha.dx$
$y \leftarrow y + 0.98\alpha.dy$
end
Where:
J: Jacobian of F.
λ: Penalty parameter.
ε: Desired Accuracy.
$\alpha \in [0,1\rangle$

3 Results

Table 2 shows the results of the execution of the programs in Matlab 19.0, obtaining the optimal values for β, l and W in 52 iterations, leaving the theoretical infrared radiation well defined for the Costa Rica station.

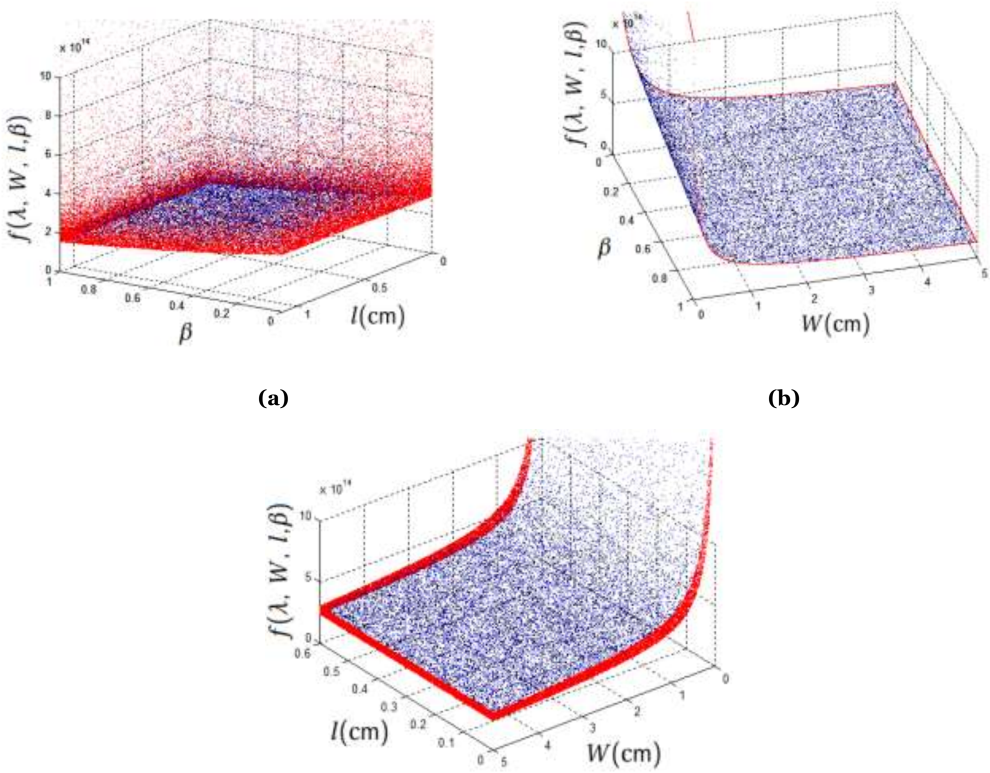
Table 2. Newton's method.

Input data	Optimum values
$L = [0,0,0], U = [1.0,1.5,5.0], \zeta = [0.3,0,4,3.9,1], \mu = 1$	$\beta = 0.9499$, $l = 1.0889$ cm y $W = 3.8342$

Table 3. Theoretical model of attenuation of infrared solar radiation for clear skies, (Costa Rica station) based on Newton's method.

Wavelength	Attenuation coefficients			Theoretical model
λ(nm)	β	l(cm)	W(cm)	$I_\lambda(\lambda, W, l, \beta)$
2100 nm *a 4000* nm	0.9499	1.0889	3.8342	$I_{o\lambda}E_o e^{-\frac{m_a}{\left[\lambda^4\left(115.6406-\frac{1.335}{\lambda^2}\right)\right]}-\frac{0.2385k_{w\lambda}3.8m_r}{(1+20.07k_{w\lambda}3.8m_r)^{0.45}}-\frac{1.41k_{g\lambda}m_a}{(1+118.93k_{g\lambda}m_a)^{0.45}}-k_{o\lambda}1.1m_o-0}$

From Table 3 it can be seen that the absorption coefficients in the spectrum (2100 nm to 4000 nm) of infrared radiation are essentially attenuated by ozone absorption l=1.0889 cm, aerosol absorption β=0.9499, with the predominant absorption by water vapor W=3.8342 cm.

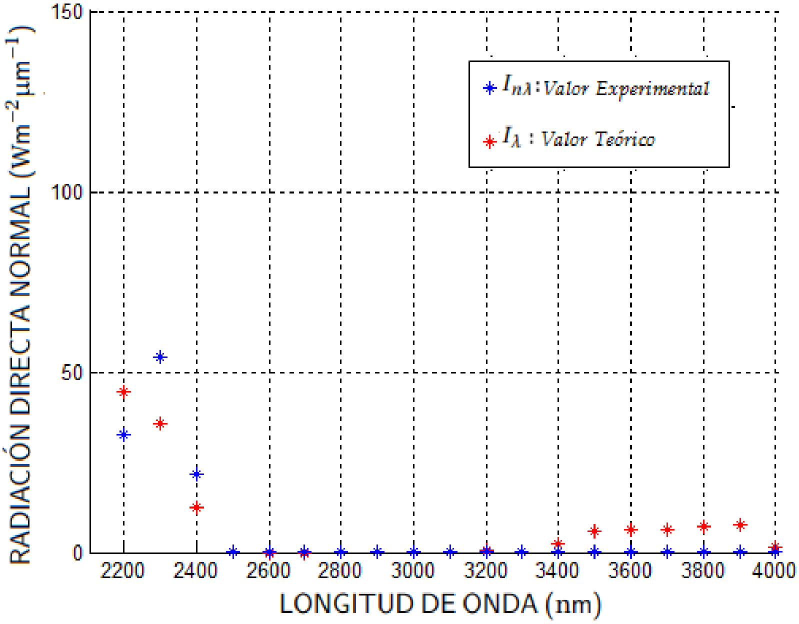


(a) (b)

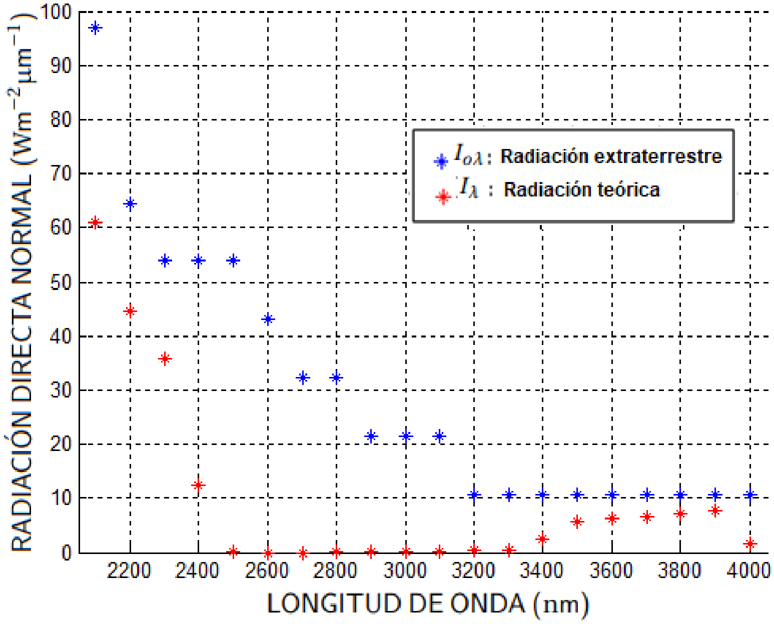
(c)

Figure 2. Representations of $f(\lambda, W, l, \beta)$ a) as a function of l and β, b) as a function of W and β and c) as a function of W and l in Matlab 19.0

From Figure 2a it is verified that $f(\lambda, W, l, \beta)$ for optimized β, is minimized when l=1.0889 cm and β=0.9499.
From Figure 2b it is verified that $f(\lambda, W, l, \beta)$ for l optimized, is minimized when W=3.8342 cm and β=0.9499.
From Figure 2c it is verified that $f(\lambda, W, l, \beta)$ for optimized β, is minimized when l=1.0889 cm and W=3.8342 cm.



(a)



(b)

Figure 3. Comparison plots of $I_\lambda(\lambda, w, l, \beta)$ with a) $I_{n\lambda}$, b) $I_{o\lambda}$.in Matlab 19.0.

From Figure 3 a, we see that the values of $I_\lambda(\lambda, w, l, \beta)$ approach $I_{n\lambda}(\lambda, W, l, \beta)$.
From Figure 3 b, it can be seen that the values of $I_{o\lambda}$ are attenuated by the ozone layer, aerosols and predominantly by water vapor, with $I_\lambda(\lambda, W, l, \beta)$ computed.

4 Discussion

Using Newton's method in Matlab 19.0, it was possible to minimize an infrared radiation function (290 nm to 4000) determining W, l and β for the Costa Rica station, so the function is well defined.
It is suggested to establish Newton's method for the automation of infrared solar radiation to obtain better results.

5 Conclusion

It is concluded according to the results that the infrared radiation data of Costa Rica corresponds to an atmosphere of cloudy white sky without the presence of an ozone hole with a considerable presence of water vapor.

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