EOQ Inventory Model for Perishable Items with Price Dependent Demand and Preservation Technique under Upstream and Downstream Trade Credit

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Perishable items are typically fresh food that has a relatively short lifetime. To increase the lifetime of perishable items preservation techniques are prone. Suppliers consider a two-level partial trade credit model to sell the goods before it deteriorates to minimize the revenue loss. The two-level trade credits are a) Supplier proposes that the retailer pay a portion of the order in cash when it is delivered, and then offers a short-term interest-free loan for the balance and b) Retailers offer partial delay in payment to their customers. The present research article investigates the analysis of the EOQ inventory model on the perishable items with a price-dependent demand under upstream and downstream partial trade credit. Economic Order Quantity is calculated for the propound inventory model using the given parameters.

Keywords: Inventory, Preservation technique, Trade credit.

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1 Introduction

In the current scenario, trade credit plays a vital role in the inventory model to increase the demand which reduces the selling price of the product. Trade credit means ‘delay in payment’ or ‘Buy now and pay later’. The suppliers provide a trade credit period to the retailers, where retailers can pay the sum without interest during that period. Goyal [1] first proposed an inventory control model under delay in the period, where Goyal [1] assumed that manufacturers provide a trade credit period to vendors, but vendors do not provide credit periods to customers. From this point of view, many researchers developed an inventory model with different scenarios. S Das et al. [2] developed a partial trade credit inventory model with reliability to solve nonlinear optimal production problems through the Taylors series approximation method. S Sahu et al. [3] proposed trade credit model for deteriorating items in which preservation technology helps in the optimization the total profit. LY Ouyard et al. [4] analyzed a production inventory model with an arithmetic-geometric mean inequality approach to finding the optimal production policy. LEC Barron et al. [5] developed an EOQ inventory model with nonlinear stock-dependent demand under trade credit, which determines the optimal order quantity and ending inventory level to maximize the retailer’s total profit.

On the inventory model, Huang et al. [6] established a two-level credit period, in which suppliers provide a credit period to retailers in exchange for retailers providing a credit period to customers. Initially Huang [6] consider that retailer’s credit period is less than the supplier’s credit period. Later developed on various trade credit on different circumstances. Suppliers and retailers offer various trade credits to the retailers and customers like partial and complete trade credit. S Chen et al. [7] proposed an inventory model for time-varying deteriorating items that are solved using discount cash flow analysis based on two-level trade credit.

Perishable items like fruits, vegetables and meats get deteriorated after a period of time. To minimize the deterioration rate, various preservation techniques are adapted. Hsu et al. [8] introduced perishable techniques on the perishable inventory model. Later Huang analyzed an inventory model under two-level trade credit and preservation Techniques. S Das et al. [9]; Dye et al. [10]; U Mishra et al. [11] and AA Shaikh et al. [12] developed various inventory models under preservation technique. Under preservation techniques and trade credit, Sahu et al.[3] offered a complete backlog inventory model to maximize the total profit. Rahman et al.[13] developed a preservation strategy for deteriorating item with

hybrid price and stock-dependent demand under advance payment.
The structure of the paper is observed as: In section 2, assumption and notation are briefly explored. The inventory model with two-level trade credit is mathematically formulated in Section 3. Section 4 describes the theoretical proof to find the optimal solution of the inventory model. Section 5 concludes with future scope.

2 Assumptions

The following assumptions would be used to develop this model:

1. The demand $D(p) = a - bp$ is a function of price-dependent demand where a denotes the demand scale and b denotes the price-sensitive parameters. D and D(p) are the same in this model.
2. Instantaneous replenishment
3. Lead time is zero
4. No shortage
5. Suppliers offer retailers upstream partial trade credit of R so that they can pay a partial sum of ζ when placing an order and settle the remaining cost within the credit period.
6. Retailers offer customers downstream partial trade credit of C so that they can pay a portion of the cost η when placing an order and settle the balance within the credit period.

3 Mathematical Formulations

The differential equation that reflects the instantaneous condition of inventory through time is $(0, T)$

$$\frac{dI(t)}{dt} + \vartheta I(t) = -D, \qquad 0 \leq t \leq T \tag{39.1}$$

Initial Inventory at t=0, $I(t) = I_0, \;\; I(t) = I_0 e^{-\vartheta t} + \frac{D}{\vartheta}[e^{-\vartheta t} - 1]$
At $t = T, I(T) = 0$, we get

$$I_0 = Q = \frac{D}{\vartheta}[e^{-\vartheta T} - 1], \qquad 0 \leq t \leq T \tag{39.2}$$

Sub in equation (2) we get,

$$I(t) = \frac{D}{\vartheta}[e^{-\vartheta(T-t)} - 1], \qquad 0 \le t \le T \tag{39.3}$$

Following costs are considered in defining a proposed inventory model. They are

1. Ordering cost is C_o
2. Sales revenue
$$C_{sr} = p\int_0^T Ddt = pDT \tag{39.4}$$
3. Holding cost
$$C_h = H_c\int_0^T I(t)dt = \frac{H_cD}{\vartheta^2}[e^{\vartheta T} - \vartheta T - 1] \tag{39.5}$$
4. Deterioration cost
$$\begin{aligned} C_d = Q - D &= \frac{D}{\vartheta}[e^{-\vartheta T} - 1] - D \\ &= D[\frac{e^{-\vartheta T}}{\vartheta} - \frac{1}{\vartheta} - 1] \end{aligned} \tag{39.6}$$
5. Preservation cost = ξT
6. Interest earned and Interest paid varies based on credit period C and R. They are $C \le R$ and $C \ge R$.

Case 1: $C \le R$

Customer's trade credit period offered by the manufacturer is less than manufacturers trade credit offered by the suppliers is again sub-classified as according to R, C, $T + C$ as $R \le T$, $R \le T + C$ and $R \ge T + C$.

Case 1.1: $C \le T$

Retailers gain interest from two parts: Immediate payment(from 0 to R) and delayed payment(from C to R) and Retailers have to finance all items sold after R in immediate payment and R-C‘ in delayed payment, they are

$$\text{Interest earned} = sI_e[\frac{\zeta DR^2}{2} + \frac{(1-\zeta)D(R-C)^2}{2}] \tag{39.7}$$

$$\text{Interest paid} = cI_p[\frac{(1-\eta)^2DT^2}{2} + \frac{\zeta D(T-R)^2}{2} + \frac{(1-\zeta)D(T+C-R)^2}{2}] \tag{39.8}$$

$$\text{Total Profit(TP)} = \frac{1}{T}[C_{sr} + IE - [C_o + C_h + C_d + PC]] \tag{39.9}$$

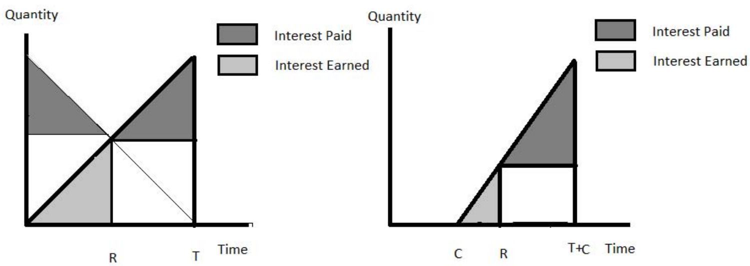

Figure 1: Instant payment and Delayed payment in $C \leq R$ and $R \leq T$

$$TP_{1.1}(T) = \frac{1}{T}[pDT + sI_e[\frac{\zeta DR^2}{2} + \frac{(1-\zeta)D(R-C)^2}{2}] - (C_o + \frac{H_cD}{2\vartheta^2}[e^{\vartheta(T-2T_1)}] + D[\frac{e^{-\vartheta T}}{\vartheta} - \frac{1}{\vartheta} - 1] + \xi + cI_p[\frac{(1-\eta)^2DT^2}{2} + \frac{\zeta D(T-R)^2}{2} + \frac{(1-\zeta)D(T+C-R)^2}{2}])] \tag{39.10}$$

Case 1.2: $R \leq T + C$

Retailers gain interest from two parts: Immediate payment(from 0 to R) and delayed payment(from C to R) and Retailers have to finance all item sold after R in immediate payment and R-C in delayed payment, they are

$$\text{Interest earned} = sI_e[\frac{\zeta DS^2}{2} + \frac{(1-\zeta)D(R-C)^2}{2}] \tag{39.11}$$

$$\text{Interest paid} = cI_p[\frac{(1-\eta)^2DT^2}{2} + \frac{\zeta D(T-R)^2}{2} + (1-\eta)DT(T+C) + \frac{(1-\zeta)D(T+C-R)^2}{2}] \tag{39.12}$$

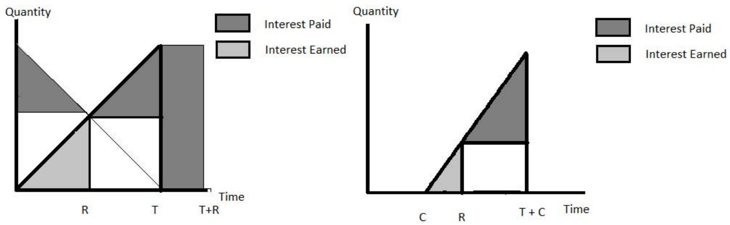

Figure 2: Instant payment and Delayed payment in $C \leq R$ and $R \leq T+C$

$$\begin{aligned} TP_{1.2}(T) = &\frac{1}{T}[pDT + sI_e[\frac{\zeta DS^2}{2} + \frac{(1-\zeta)D(R-C)^2}{2}] \\ &-(C_o + DpT + \frac{H_cD}{\vartheta^2}[e^{\vartheta T} - \vartheta T - 1] + D[\frac{e^{-\vartheta T}}{\vartheta} - \frac{1}{\vartheta} - 1] \\ &+ \xi + cI_p[\frac{(1-\eta)^2DT^2}{2} + \frac{\zeta D(T-R)^2}{2} \\ &+ (1-\eta)DT(T+C) + \frac{(1-\zeta)D(T+C-R)^2}{2}]] \end{aligned} \tag{39.13}$$

Case 1.3: $R \geq T + C$

Retailers gain interest from two parts: Immediate payment (from 0 to R) and delayed payment (from C to R) and Retailers have to finance all item sold after R in immediate payment, they are

$$\text{Interest earned} = sI_e[\frac{\zeta DT^2}{2} + \frac{(1-\zeta)DT^2}{2} + \zeta DT(R-T) + (1-\zeta)DT(R-T-C)] \quad (39.14)$$

$$\text{Interest paid} = cI_p[\frac{(1-\eta)^2 DT^2}{2}] \quad (39.15)$$

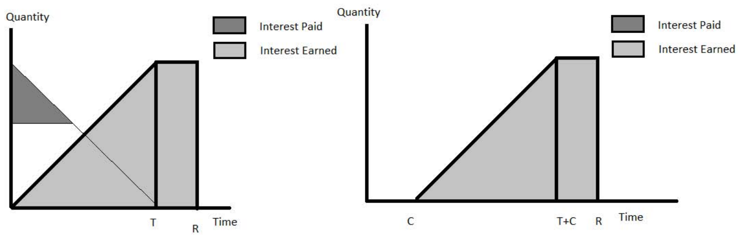

Figure 3: Instant payment and Delayed payment in $C \leq R$ and $R \geq T+C$

$$TP_{1.3}(T) = \frac{1}{T}[pDT + sI_e[\frac{\zeta DT^2}{2} + \zeta DT(R-T) + \frac{(1-\zeta)DT^2}{2} + (1-\zeta)DT(R-T-C)] - (C_o + DpT + \frac{H_c D}{\vartheta^2}[e^{\vartheta T} - \vartheta T - 1] + D[\frac{e^{-\vartheta T}}{\vartheta} - \frac{1}{\vartheta} - 1] + \xi + cI_p[\frac{(1-\eta)^2 DT^2}{2}] \quad (39.16)$$

Case 2: $C \geq R$

Customers trade credit period from manufacturer is greater than manufacturers trade credit from the suppliers is sub-classified as $R \leq T$ and $R \geq T$

Case 2.1: $R \leq T$

Retailers gain interest from Immediate payment (i.e)from 0 to R and Retailers have to finance the initial amount payment made by suppliers, all items sold after R in immediate payment and (R to τ T+C) in delayed payment, they are

$$\text{Interest earned} = sI_e[\frac{\zeta DT^2}{2}] \tag{39.17}$$

$$\text{Interest paid} = cI_p[\frac{(1-\eta)^2 DT^2}{2} + (1-\eta)DT(T+C) + (1-\zeta)D(\frac{T^2}{2} + T(C-R))] \tag{39.18}$$

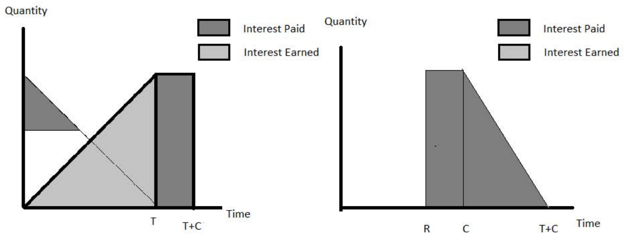

Figure 4: Instant payment and Delayed payment in $C \geq R$ and $R \leq T$

$$\begin{aligned} TP_{2.1}(T) =& \frac{1}{T}[pDT + sI_e[\frac{\zeta DT^2}{2}] - (C_o + DpT \\ &+ \frac{H_c D}{\vartheta^2}[e^{\vartheta T} - \vartheta T - 1] + D[\frac{e^{-\vartheta T}}{\vartheta} - \frac{1}{\vartheta} - 1] \\ &+ \xi + sI_p[\frac{(1-\eta)^2 DT^2}{2} + (1-\eta)DT(T+C) \\ &+ (1-\zeta)D(\frac{T^2}{2} + T(C-R))] \end{aligned} \tag{39.19}$$

Case 2.2: $R \geq T$

Retailers gain interest from Immediate payment (i.e)from 0 to R and Retailers have to finance the initial amount payment made by suppliers, all items sold after R in immediate payment and (R to T+C) in delayed payment, they are

$$\text{Interest earned} = sI_e[\frac{\zeta DS^2}{2}] \tag{39.20}$$

$$\text{Interest paid} = cI_p[\frac{(1-\eta)^2DT^2}{2} + (1-\eta)DT(T+C) + (1-\zeta)D(\frac{T^2}{2} + T(C-R))] \tag{39.21}$$

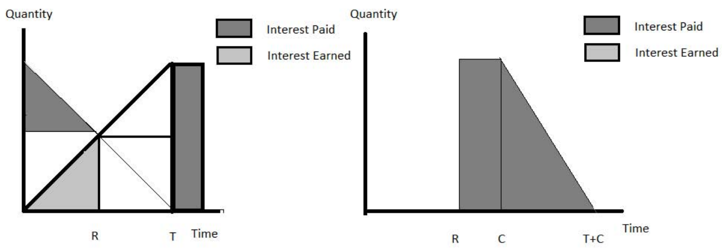

Figure 5: Instant payment and Delayed payment in $C \geq R$ and $R \geq T$

$$TP_{2.2}(T) = \frac{1}{T}[pDT + sI_e[\frac{\zeta DS^2}{2}] - (C_o + DpT + \frac{H_cD}{\vartheta^2}[e^{\vartheta T} - \vartheta T - 1] + D[\frac{e^{-\vartheta T}}{\vartheta} - \frac{1}{\vartheta} - 1] + \xi + cI_p[\frac{(1-\eta)^2DT^2}{2} + (1-\eta)DT(T+C) + (1-\zeta)D(\frac{T^2}{2} + T(C-R))] \tag{39.22}$$

4 Theoretical Proof of Optimal Solution

The necessary and sufficient condition for determining the optimal solution for all the cases of trade credit. A function $q(x) = \frac{f(x)}{g(x)}$ is Pseudo concave. if f(x) is positive, differentiable and Convex and g(x) is non-negative, differentiable and Concave by Cambini and Martein's (2009) theorem 3.2.10. Cambini and Martein (2009) Theorem 3.2.9 states that q(x) is strictly Pseudo- convex if f(x) is strictly concave. Applying this to prove the TP(T) is strictly concave. Where f(x) varies for each case but $g(x) = T$, which is differentiable and concave. Now following theorem state that $f_i(x)$ is concave.

Theorem 4.1.1

(a) $TP_{1.1}(T)$ is strictly Pseudo concave function in T, and hence exist a unique maximum solution T_1^*

(b) if $R \leq T_1^*$, then $TP_{1.1}(T)$ subject to $R \leq T$ is maximized at T_1^*

(c) if $R \geq T_1^*$, then $TP_{1.1}(T)$ subject to $R \leq T$ is maximized at R

$$\begin{aligned} f_1 =& [pDT + sI_e[\frac{\zeta DS^2}{2} + \frac{(1-\zeta)D(R-C)^2}{2}] - (C_o \\ &+ \frac{H_cD}{2\vartheta^2}[e^{\vartheta(T-2T_1)}] + D[\frac{e^{-\vartheta T}}{\vartheta} - \frac{1}{\vartheta} - 1] + \xi \\ &+ cI_p[\frac{(1-\eta)^2DT^2}{2} + \frac{\zeta D(T-R)^2}{2} \\ &+ \frac{(1-\zeta)D(T+C-R)^2}{2}])] \end{aligned} \tag{39.23}$$

Taking $f_1(T)$ first and second-order differentiation into consideration

$$\begin{aligned} \frac{df_1}{dT} =& Dp - (\frac{H_cD}{\vartheta^2}[\frac{e^{\vartheta T}}{\vartheta} - \vartheta] - D[\frac{e^{-\vartheta T}}{\vartheta^2}] \\ &+ cI_p[(1-\eta)^2DT + \zeta D(T-R) \\ &+ (1-\zeta)D(T+C-R)]) \end{aligned} \tag{39.24}$$

$$\begin{aligned} \frac{d^2f_1}{d^2T} =& -(\frac{H_cDe^{\vartheta T}}{\vartheta^4} + D[\frac{e^{-\vartheta T}}{\vartheta^3}] \\ &+ cI_p[(1-\eta)^2D + \zeta D + (1-\zeta)D]) < 0 \end{aligned} \tag{39.25}$$

As a result of the pseudo concave function of $TP_{1.1}(T) = f_1(T)/g_1(T)$ proof the (a) of theorem 4.1.1 and Result of (b)and (c) are directly preceded by (a) of theorem 4.1.1

To find T_1^*, we equate $\frac{dTP_{1.1}(T)}{dT} = 0$

$$\begin{aligned}\frac{dTP_{1.1}(T)}{dT} =& -[pDT + sI_e[\frac{\zeta DS^2}{2} + \frac{(1-\zeta)D(R-C)^2}{2}] \\ & - (C_o + \frac{H_c D}{2\vartheta^2}[e^{\vartheta(T-2T_1)}] + D[\frac{e^{-\vartheta T}}{\vartheta} - \frac{1}{\vartheta} - 1] + \xi \\ & + cI_p[\frac{(1-\eta)^2 DT^2}{2} + \frac{\zeta D(T-R)^2}{2} \\ & + \frac{(1-\zeta)D(T+C-R)^2}{2}])] + Dp - (\frac{H_c D}{\vartheta^2}[\frac{e^{\vartheta T}}{\vartheta} - \vartheta] - D[\frac{e^{-\vartheta T}}{\vartheta^2}] \\ & + cI_p[(1-\eta)^2 DT + \zeta D(T-R) \\ & + (1-\zeta)D(T+C-R)]) = 0 \end{aligned} \tag{39.26}$$

Equation (26) proves that there is a unique T_1^*. if $R \leq T_1^*$, then $TP_{1.1}(T)$ is maximized at T_1^* otherwise, it is maximum at R

Theorem 4.1.2

(a) $TP_{1.2}(T)$ is strictly Pseudo concave function in T, and hence exist a unique maximum solution T_2^*

(b) if $R \leq T_2^* + C$, then $TP_{1.2}(T)$ subject to $R \leq T + C$ is maximized at T_2^*

(c) if $R \geq T_2^* + C$, then $TP_{1.2}(T)$ subject to $R \leq T + C$ is maximized at $R - C$

$$\begin{aligned} f_2 =& [DpT + sI_e[\frac{\zeta DS^2}{2} + \frac{(1-\zeta)D(R-C)^2}{2}] \\ & - (C_o + \frac{H_c D}{\vartheta^2}[e^{\vartheta T} - \vartheta T - 1] + D[\frac{e^{-\vartheta T}}{\vartheta} - \frac{1}{\vartheta} - 1] + \xi \\ & + cI_p[\frac{(1-\eta)^2 DT^2}{2} + \frac{\zeta D(T-R)^2}{2} + (1-\eta)DT(T+C) \\ & + \frac{(1-\zeta)D(T+C-R)^2}{2}])] \end{aligned} \tag{39.27}$$

Taking $f_2(T)$ first and second-order differentiation into consideration

$$\frac{df_2}{dT} = Dp - (\frac{H_c D}{\vartheta^2}[\frac{e^{\vartheta T}}{\vartheta} - \vartheta - D[\frac{e^{-\vartheta T}}{\vartheta^2}] + cI_p[(1-\eta)^2 DT$$
$$+ \zeta D(T-R) + (1-\zeta)D(T+C-R) + (1-\eta)D(2T+C)]) \quad (39.28)$$
$$\frac{d^2 f_2}{d^2 T} = -(\frac{H_c D e^{\vartheta T}}{\vartheta^4} + D[\frac{e^{-\vartheta T}}{\vartheta^3}] + cI_p[(1-\eta)^2 D$$
$$+ \zeta D + (1-\zeta)D + 2(1-\eta)D]) < 0 \quad (39.29)$$

As a results of the pseudo concave function of $TP_{1.2}(T) = f_2(T)/g_2(T)$ proof the (a) of theorem 4.1.2 and Result of (b)and (c) are directly preceded by (a) of theorem 4.1.2

To find T_2^*, we equate $\frac{dTP_{1.2}(T)}{dT} = 0$

$$\frac{dTP_{1.2}(T)}{dT} = -[DpT + sI_e[\frac{\zeta DS^2}{2} + \frac{(1-\zeta)D(R-C)^2}{2}]$$
$$-(C_o + \frac{H_c D}{\vartheta^2}[e^{\vartheta T} - \vartheta T - 1]$$
$$+ D[\frac{e^{-\vartheta T}}{\vartheta} - \frac{1}{\vartheta} - 1] + \xi + cI_p[\frac{(1-\eta)^2 DT^2}{2}$$
$$+ \frac{\zeta D(T-R)^2}{2} + (1-\eta)DT(T+C)$$
$$+ \frac{(1-\zeta)D(T+C-R)^2}{2}])] + Dp - (\frac{H_c D}{\vartheta^2}[\frac{e^{\vartheta T}}{\vartheta} - \vartheta] - D[\frac{e^{-\vartheta T}}{\vartheta^2}]$$
$$+ cI_p[(1-\eta)^2 DT + \zeta D(T-R)$$
$$+ (1-\zeta)D(T+C-R) + (1-\eta)D(2T+C)]) = 0 \quad (39.30)$$

Equation (30) proves that their is a unique T_2^*. if $R \leq T_2^* + C$, then $TP_{1.2}(T)$ is maximized at T_2^* otherwise it is maximum at $R - C$

Theorem 4.1.3

(a) $TP_{1.3}(T)$ is strictly Pseudo concave function in T, and hence exist a unique maximum solution T_3^*

(b) if $R \leq T_3^* + C$, then $TP_{1.3}(T)$ subject to $R \geq T + C$ is maximized at T_3^*

(c) if $R \geq T_3^* + C$, then $TP_{1.3}(T)$ subject to $R \geq T + C$ is maximized at $R - C$

$$
\begin{aligned}
f_3 =& [DpT + sI_e[\frac{\zeta DT^2}{2} + \frac{(1-\zeta)DT^2}{2} \\
& + \zeta DT(R-T) + (1-\zeta)DT(R-T-C)] \\
& - (C_o + \frac{H_c D}{\vartheta^2}[e^{\vartheta T} - \vartheta T - 1] + D[\frac{e^{-\vartheta T}}{\vartheta} - \frac{1}{\vartheta} - 1] + \xi \\
& + cI_p[\frac{(1-\eta)^2 DT^2}{2})]
\end{aligned} \tag{39.31}
$$

Taking $f_3(T)$ first and second-order differentiation into consideration

$$
\begin{aligned}
\frac{df_3}{dT} =& Dp + sI_e[\zeta DT + (1-\zeta)DT + \zeta D(R-2T) \\
& + (1-\zeta)D(R-2T-C)] - (\frac{H_c D}{\vartheta^2}[\frac{e^{\vartheta T}}{\vartheta} - \vartheta] \\
& + D[\frac{e^{-\vartheta T}}{\vartheta^3}] + cI_p[(1-\eta)^2 DT])
\end{aligned} \tag{39.32}
$$

$$
\begin{aligned}
\frac{d^2 f_3}{d^2 T} =& sI_e[\zeta D + (1-\zeta)D - 2\zeta D - 2(1-\zeta)D] \\
& - (\frac{H_c De^{\vartheta T}}{\vartheta^4} + \frac{D}{\vartheta}[\frac{e^{\vartheta T}}{\vartheta} - \frac{e^{2\vartheta T}}{4\vartheta^2}] + cI_p[(1-\eta)^2 DT]) < 0
\end{aligned} \tag{39.33}
$$

As a result of the pseudo concave function of $TP_{1.3}(T) = f_3(T)/g_3(T)$ proof the (a) of theorem 4.1.3 and Result of (b)and (c) are directly preceded by (a) of theorem 4.1.3

To find T_3^*, we equate $\frac{dTP_{1.3}(T)}{dT} = 0$

$$\begin{aligned}\frac{dTP_{1.3}(T)}{dT} =& -[DpT + sI_e[\frac{\zeta DT^2}{2} + \frac{(1-\zeta)DT^2}{2} \\ &+ \zeta DT(R-T) + (1-\zeta)DT(R-T-C)] \\ &-(C_o + \frac{H_cD}{\vartheta^2}[e^{\vartheta T} - \vartheta T - 1] + D[\frac{e^{-\vartheta T}}{\vartheta} - \frac{1}{\vartheta} - 1] + \xi \\ &+ cI_p[\frac{(1-\eta)^2DT^2}{2})] + Dp + sI_e[\zeta DT + (1-\zeta)DT + \zeta D(R-2T) \\ &+ (1-\zeta)D(R-2T-C)] - (\frac{H_cD}{\vartheta^2}[\frac{e^{\vartheta T}}{\vartheta} - \vartheta] \\ &+ D[\frac{e^{-\vartheta T}}{\vartheta^3}] + cI_p[(1-\eta)^2DT]) = 0 \end{aligned} \tag{39.34}$$

Equation (34) proves that there is a unique T_3^*. if $R \leq T_3^* + C$, then $TP_{1.3}(T)$ is maximized at T_3^* otherwise, it is maximum at $R - C$

Theorem 4.2.1

(a) $TP_{2.1}(T)$ is strictly Pseudo concave function in T, and hence exist a unique maximum solution T_4^*

(b) if $R \leq T_4^*$, then $TP_{2.1}(T)$ subject to $R \leq T$ is maximized at T_4^*

(c) if $R \geq T_4^*$, then $TP_{2.1}(T)$ subject to $R \leq T$ is maximized at R

$$\begin{aligned} f_4 =& [DpT + sI_e[\frac{\zeta DT^2}{2}] \\ &-(C_o + \frac{H_cD}{\vartheta^2}[e^{\vartheta T} - \vartheta T - 1] + D[\frac{e^{-\vartheta T}}{\vartheta^3}] \\ &+ \xi + cI_p[\frac{(1-\eta)^2DT^2}{2} + (1-\eta)DT(T+C) \\ &+ (1-\zeta)D(\frac{T^2}{2} + T(C-R))] \end{aligned} \tag{39.35}$$

Taking $f_4(T)$ first and second-order differentiation into consideration

$$\begin{aligned}\frac{df_4}{dT} =& Dp + sI_e[\zeta DT] - (\frac{H_c D}{\vartheta^2}[\frac{e^{\vartheta T}}{\vartheta} \\ & - \vartheta] - D[\frac{e^{-\vartheta T}}{\vartheta^2}] + cI_p[(1-\eta)^2 DT \\ & + (1-\eta)D(2T + C) + (1-\zeta)D[T + (C-R)]]) \end{aligned} \tag{39.36}$$

$$\begin{aligned}\frac{d^2 f_4}{d^2 T} =& sI_e[\zeta D] - (\frac{H_c D e^{\vartheta T}}{\vartheta^4} + \frac{D}{\vartheta}[\frac{e^{\vartheta T}}{\vartheta} - \frac{e^{2\vartheta T}}{4\vartheta^2}] \\ & + cI_p[(1-\eta)^2 D + 2(1-\eta)D + +(1-\zeta)D]) < 0 \end{aligned} \tag{39.37}$$

As a result, the pseudo concave function of $TP_{2.1}(T) = f_4(T)/g_4(T)$F. This proof the (a) of theorem 4.2.1 and Result of (b)and (c) are directly preceded by (a) of theorem 4.2.1

To find T_4^*, we equate $\frac{dTP_{2.1}(T)}{dT} = 0$

$$\begin{aligned}\frac{dTP_{2.1}(T)}{dT} =& -[DpT + sI_e[\frac{\zeta DT^2}{2}] \\ & - (C_o + \frac{H_c D}{\vartheta^2}[e^{\vartheta T} - \vartheta T - 1] + D[\frac{e^{-\vartheta T}}{\vartheta^3}] \\ & + \xi + cI_p[\frac{(1-\eta)^2 DT^2}{2} + (1-\eta)DT(T + C) \\ & + (1-\zeta)D(\frac{T^2}{2} + T(C-R))] + Dp + sI_e[\zeta DT] - (\frac{H_c D}{\vartheta^2}[\frac{e^{\vartheta T}}{\vartheta} \\ & - \vartheta] - D[\frac{e^{-\vartheta T}}{\vartheta^2}] + cI_p[(1-\eta)^2 DT \\ & + (1-\eta)D(2T + C) + (1-\zeta)D[T + (C-R)]]) = 0 \end{aligned} \tag{39.38}$$

Equation (38) proves that there is a unique T_4^*. if $R \leq T_4^*$, then $TP_{2.1}(T)$ is maximized at T_4^* otherwise, it is maximum at R

Theorem 4.2.2

(a) $TP_{2.2}(T)$ is strictly Pseudo concave function in T, and hence exist a unique maximum solution T_5^*

(b) if $R \geq T_5^*$, then $TP_{2.2}(T)$ subject to $s \leq T$ is maximized at T_5^*

(c) if $R \leq T_5^*$, then $TP_{2.2}(T)$ subject to $R \leq T$ is maximized at R

$$\begin{aligned} f_5 =& [DpT + sI_e[\frac{\zeta DS^2}{2}] - (C_o \\ &+ \frac{H_c D}{\vartheta^2}[e^{\vartheta T} - \vartheta T - 1] + D[\frac{e^{-\vartheta T}}{\vartheta} - \frac{1}{\vartheta} - 1] + \xi \\ &+ cI_p[\frac{(1-\eta)^2 DT^2}{2} + (1-\eta)DT(T+C) \\ &+ (1-\zeta)D(\frac{T^2}{2} + T(C-R))] \end{aligned} \tag{39.39}$$

Taking $f_5(T)$ first and second-order differentiation into consideration

$$\begin{aligned} \frac{df_5}{dT} =& Dp - (\frac{H_c D}{\vartheta^2}[\frac{e^{\vartheta T}}{\vartheta} - \vartheta] - D[\frac{e^{-\vartheta T}}{\vartheta^2}] \\ &+ cI_p[(1-\eta)^2 DT + (1-\eta)D(2T+C) \\ &+ (1-\zeta)D[T + (C-R)]) \end{aligned} \tag{39.40}$$

$$\begin{aligned} \frac{d^2 f_5}{d^2 T} =& -(\frac{H_c De^{\vartheta T}}{\vartheta^4} + D[\frac{e^{-\vartheta T}}{\vartheta^3}] \\ &+ cI_p[(1-\eta)^2 D + 2(1-\eta)D + (1-\zeta)D]) < 0 \end{aligned} \tag{39.41}$$

As a results, the pseudo concave function of $TP_{2.2}(T) = f_5(T)/g_5(T)$. This proof the (a) of theorem 4.2.2 and Result of (b)and (c) are directly preceded by (a) of theorem 4.2.2

To find T_5^*, we equate $\frac{dTP_{2.2}(T)}{dT} = 0$

$$\begin{aligned}\frac{dTP_{2.2}(T)}{dT} =& - [DpT + sI_e[\frac{\zeta DS^2}{2}] - (C_o \\ &+ \frac{H_c D}{\vartheta^2}[e^{\vartheta T} - \vartheta T - 1] + D[\frac{e^{-\vartheta T}}{\vartheta} - \frac{1}{\vartheta} - 1] + \xi \\ &+ cI_p[\frac{(1-\eta)^2 DT^2}{2} + (1-\eta)DT(T+C) \\ &+ (1-\zeta)D(\frac{T^2}{2} + T(C-R))] + Dp- \\ &(\frac{H_c D}{\vartheta^2}[\frac{e^{\vartheta T}}{\vartheta} - \vartheta] - D[\frac{e^{-\vartheta T}}{\vartheta^2}] \\ &+ cI_p[(1-\eta)^2 DT + (1-\eta)D(2T+R) \\ &+ (1-\zeta)D[T + (R-R)]) = 0 \end{aligned} \quad (39.42)$$

Equation (42) proves that there is a unique T_5^*. if $R \geq T_5^*$, then $TP_{2.2}(T)$ is maximized at T_5^* otherwise it is maximum at R

5 Conclusion

In this work, price dependent inventory model is discussed with preservation technology under upstream and downstream trade credit. Total profit for various cases are calculated. It is classified based on the credit period and it is subclassified based on items on hand. The optimal solution of all the sub-cases maximizes the retailer's total profit. This model can be extended for optimal ordering policy, two-level complete trade credit and so on.

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Notation	
c	Unit cost per unit time
C_d	Deterioration cost
C_o	Ordering cost
C_h	Holding cost
C_{sr}	Sales revenue
D	Demand
H_c	Inventory carrying cost
I_p	Interest paid per unit time
I_e	Interest earned per unit time
$I(t)$	Inventory level at time t, $0 \leq t \leq T$
p	Purchase cost per unit time
Q	Replenishment at time T
C	Retailers offer a trade credit period to their customer
R	Suppliers offer a trade credit period to their retailer's
T	Length of the inventory cycle
TP	Total Profit
ϑ	Deterioration rate
ξ	Preservation technique
ζ	Retailer must pay the supplier a percentage of the purchase price at the time of placing the order
$1-\zeta$	A credit period for a portion of the purchase cost is offered to retailers
η	A percent of the purchase cost that the customer as to pay to the retailers
$1-\eta$	Credit period for a portion of the purchase cost is offered to customers.