Performance Analysis of Preconditioner based Image Reconstruction in High Resolution Microwave Tomography

N. Nithya

Thiagarajar College of Engineering, Madurai, Tamilnadu, India

MSK. Manikandan

Thiagarajar College of Engineering, Madurai, Tamilnadu, India

Corresponding author: N. Nithya, Email: nithyan@student.tce.edu

The accuracy and speed of convergence in Microwave Tomography Imaging System (MwTIS) affected by ill-condition nature of coefficient matrix. Krylov subspaces based regularization methods are effectively produce the solution with ill-condition problems.The proposed work, details the preconditioner based regularization method called Enriched Conjugate Gradient Least Square (ECGLS) is good at handling ill-condition problem in MwTIS. In this paper, first, it analyzes the impact of frequency changes in the condition number of the coefficient matrix and evaluates the efficacy of ECGLS method.It achieves 37.42% of MSE at 6 reconstruction iteration for brain dataset.

Keywords: Microwave Tomography, ill-condition problem, ECGLS, Brain Imaging, Regularization.

2021. In Raju Pal & Praveen K. Shukla (eds.), *SCRS Conference Proceedings on Intelligent Systems*, 467–474. Computing & Intelligent Systems, SCRS, India.
https://doi.org/10.52458/978-93-91842-08-6-44

1 Introduction

Nowadays, the microwave based imaging system pays more attention in medical diagnosis because of its compactness and portability in pre-hospital use [1]. Microwave Tomography Imaging System (MwTIS) is a state-of-the-art activity and inspires new thoughts by its several benefits such as it uses non ionizing low power EM signals, cost effective antenna-array, low health-risk methods , portability and can fit anywhere[2,3]. The forward process and inverse process are the two important stages which played a major role in image reconstruction algorithms. The forward process digitizes the acquired data from measurement system and converts it into linear equations. Inverse process [4] is to find unknown values(x) from these linear equations.

The several iterative image reconstruction algorithms for spatial domain were proposed in 1989 onwards such as a Born iterative method (BIM) [5], Distorted Born iterative method (DBIM) [6], Gauss Newton (GN) [7] method which derived for linearizing the MwTIS reconstruction process. A regularization method is one of the steps in reconstruction algorithm to obtain a numerically stable and good approximate solution where the linear equations are ill-conditioned and underdetermined. DBIM with regularizer like L_2-ISATCS [8], L_2-IWA[9], IMATCS[10], TwIST[11] and BIM with IST [12]in outreached performance in the breast and brain imaging were tested under various scenarios. Krylov subspaces [13] have many vital roles in regularization of large-scale inverse scattering and other scientific problems. The Krylov subspace techniques like Conjugate Gradient (CG)[14] and CGLS [15] methods used in the MwTIS were started in 1995. In 2007 the Dartmouth College used CGLS with Gauss Newton algorithm for their realistic hardware configuration model. The CGLS method determines the Jacobian for updating step length and direction which proved the convergence and accuracy of the solution. It can control the number of iterations without using explicit regularizing parameter.

The main contributions of our proposed work is focuses the changes in ill-condition number make sever oscillation in CGLS method due to the changes in the frequency ranges (shifting from low frequency to high frequency). The high frequency is highly sensitive in condition number. Even though in small changes in condition numbers severely affect the solution accuracy.

This paper contains five sections. In this paper, the formulation of the proposed methods is discussed in section 2. Section 3 states the goodness of ECGLS methods. The numerical results were analyzing the work of ECGLS methods in brain phantom are reported in section 4. Finally, section 5 explains the concluded information of the proposed study.

2 Formulation of Forward Process

This section detailed the imaging domain characterization and formation of coefficient matrix (A). Forward process computes the incident field and scattered field from the object to be imaged in the imaging domain. Circular imaging domain is modeled here. The object placed in the center of imaging domain and it is discretized into N subunits (n x n). The dielectric properties of an object characterized by relative complex permittivity. The spatial distribution of relative permittivity ε_r is expressed in below equation.

$$\varepsilon_r = [\ (\varepsilon_r - \varepsilon_b)/\varepsilon_b] \quad (1)$$

ε_b is the permittivity of background medium. The object is illuminated using the plane wave by a set of antennas in different angle θ.

$$E_{inci} = E_0.exp(-K_b(x\cos\theta + ysin\theta)) \quad (2)$$

The electric field integral equation for Born Iterative Method (BIM) to governing the entire D domain is expressed as below,

$$E_{scat}(r) = k_b^2 \oint_S G(r,r')X_{object(r')}E_{inci}(r') \; dr' \; r' \text{€} S \qquad (3)$$

It estimates the scattered field data are collected from the M number of measurement points. $G(r,r')$ is the Green's function with the wave number of background medium (K_b). $X_{object(r')}$ is the unknown object . $r$is the position vector. The resultants of forward process are obtained by Method of Moment (MoM) method. Now the outcome of the forward process have been reformulated in the form of,

$$A_{(M\ X\ N)}.\ x_{NX1} = b_{MX1} \qquad (4)$$

It is similar to linear system of equation. $A_{(M\ X\ N)}$is called measurement or system coefficient matrix. $b_{MX1}$is the measured scattered field at measurement points(M). BIM reconstruction algorithm estimates the x iteratively. The dimension of A is $M \, x \, N$ and it is in underdetermined nature $M << N$. It update the measured scattered field (3) and calculated scattered field (4) based on the current estimate of the$x$ in each iteration.

3 Importance of Proposed Work

This section explained the estimating accurate image of object (x) in the imaging domain by using Krylov subspace regularization method called ECGLS[16]. Krylovsubspaces based regularization methods are produce the solution for severely ill-condition coefficient matrix (A).

Condition number $\kappa(A)$ is one of the important characteristics of solution estimation in (4). It is a ratio between the maximum and non zero singular value of the matrix A. This is a quantity is a measure of how the solution is nearer in space of linear systems. It also influences the convergence rate of the solution estimation. It leads to less accurate solution.

The large condition number oscillates the convergence and cannot reach the appropriate solution space. In the microwave tomography imaging the integrals of forward process solved by Fredholm integrals of first kind produce highly ill-conditioned ($\kappa(A) > 1$) matrix. It is unavoidable situation in the microwave tomography imaging system. It is high sensitive to operating frequency of imaging domain. So that, the proposed work is analyze the changes of $\kappa (A)$ due to the frequency of imaging system.

3.1 Role of Preconditioner

The condition number of A is bad in ill-condition problem which leads to less accurate solution. It takes more number of iteration. This problem can be solved by embedding preconditioner (P) matrix and multiplied with A used to project the solution in new dimension in each iteration with minimum matrix vector multiplication.

$$P^{-1}.A.\ x = P^{-1}.b \qquad (5)$$

It converts the high condition of A to low condition number. The low condition number gives the solution with high accuracy. In this paper, the role of preconditioner based Krylov subspace regularization method in microwave imaging is a unique feature. The importance of this method explained in below sections.

3.2 Goodness of ECGLS

CGLS, GMRES and MRNSD are the methods in family of Krylov subspace. CGLS and MRNSD methods used in the microwave imaging system and effectively produce the results by solving ill-posed and ill-condition problems. ECGLS method computes the solution for (5). Compared to CGLS method, uniqueness of ECGLS is stated here.

- QR-decomposition based preconditioned (P) matrix retrieve small singular values in the solution (x_k) estimation.

$$x_k = x_{k-1} + \alpha_{k-1} d_{k-1} + P_{k-1}.b_{k-1} \quad (6)$$

- The solution x is projected into the span of right hand side (span$\{A^T A, A^T A b^\delta\}$) which handle the perturbation (b + Δb) in Right Hand Side (R.H.S) of equation (4) drastically reduces the number of iteration.
- The discrepancy principal (d_k) associated with the error value used as the termination criteria protect the x_k by discretization error.

$$d_{k+1} := d_{k+1} - V_{k+1}\tilde{y}_{k+1} \quad (7)$$

Here, V_{k+1},$\alpha_{k-1}$are direction vector , step length and tuning vectors respectively.$\tilde{y}_{k+1}$ is the step length of enriched subspace*(Q)*. The detailed procedure of ECGLS is in [16].

4 Results and Discussions

The analysis is made to find the impact of frequency changes in the condition number of the coefficient matrix (A) and the goodness of the proposed ECGLS method in the BIM reconstruction algorithm. The study proceeds in the aspect of:

- Estimate the condition number *κ(A)* of coefficient matrix due to the frequency variations. Measurements are done on three frequency range such as 0.8 GHz, 1 GHz and 1.2 GHz.
- Estimate the perturbation ratio in the Right side of linear equation and estimated solution
- Analyzing the performance of the ECGLS method and CGLS
- The 2D brain data model is considered for the experiment.

4.1 Simulation Setup and Dataset

The circular measurement system is configured with the radius 12 cm. The 16 propagation points are placed on the circle boundary with equally angle difference. The brain phantom data is taken from [17]. The size of 2D brain slab is 212 x 212. The size of the pixel in the imaging domain is 1mm size.

Table 1. DielectricProperties of Brain Phantom

Tissue Name	Grey Matter	White Matter	Skull	CSF	Background Medium
Dielectric value	50+j18	40+j15	13+j2	57+j26	40+13j

The dielectric properties of brain phantom are listed in Table 1. The image of the 2D brain model is shown in Fig.1.

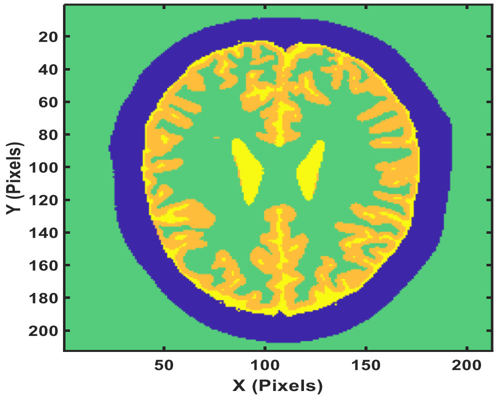

Fig.1. 2D Brain Model

4.2 Analysis of Solution Perturbation

The ill-condition problem was generally arisen in tomography algorithms. The condition number was affected by the design criterions of data acquisition system like, number of transmitting and receiving antennas, frequency, and discretization of the imaging domain. The resolution of the resultant image depends by frequency and size of pixels (discretization) in the imaging domain. High frequency gives high resolution. In the proposed work, analyze the impact of the frequency changes had affected the solution and computing time.

The results of estimated perturbation ratio of the R.H.S of linear equation and estimated solution are listed in the Table-2. The frequency is 0.8 GHz taken as the reference value. The remaining values are other frequencies (1 GHz, 1.2 GHz) estimated based that reference value using CGLS method.

Table 1. Perturbation ratio of R.H.S and Solution by Frequency variations measured using CGLS Method

Frequency (F)	Condition Number κ(A)	Relative R.H.S Error ||b+Δb||/||b||	Relative solution Error ||x+ΔX ||/||X||
0.8 GHz	23.733	-	-
1 GHz	17.0543	0.5219	0.31844
1.2 GHz	13.5503	0.8335	0.33060

It clearly depicts the small changes *A* matrix makes large variations in the results and the scattered field. Theory behind the imaging system is high frequency gives good image resolution, but CGLS show the negative effects in solution estimation.

The proposed ECGLS method performs well in this situation. The relative solution error in each iteration recorded for that frequencies, drastically reduced from 0.82605 to 0.6727 for 0.8 GHz to 1.2 GHz respectively. In CGLS it was increased from 0.7322 to 0.7783. The Fig.2 depicted the same as mentioned in above.

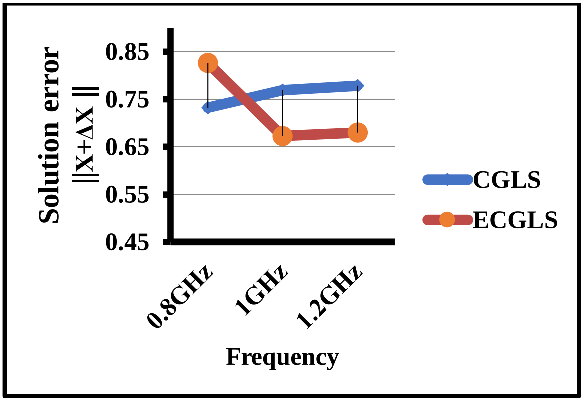

Fig.2. Comparison of CGLS and ECGLS method handling perturbation in solution due to frequency variations

4.3 Performance of Enriched CGLS

The performance of the CGLS and the ECGLS is analyzed by comparing the Mean Square Error (MSE) and number of BIM iteration for the brain phantom. In case of CGLS, 67.89 % of MSE is obtained in cases of 1.2 GHz frequency at 7 iteration steps. In case of Enriched CGLS, the MSE is about 37.42 % within the 6 iteration. The MSE fluctuates large with the increase of frequency and also required more iteration to converge in CGLS.

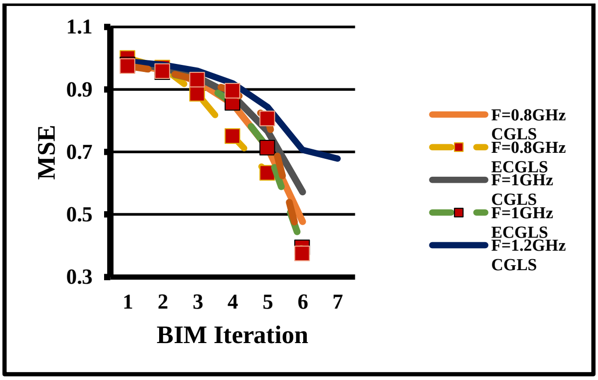

Fig. 3. Convergence of BIM algorithm using ECGLS and CGLS methods in Frequency Variations

The MSE achieved at final iteration are 47.72 %, 57.21% and 67.89 % for 0.8,1 and 1.2 GHz respectively. In case of ECGLS, the MSE is about 63.24%, 46% and 37.42 in all frequency values. These results are obtained within 5-6 iterations. The figure 3 concluded that the ECGLS algorithm provides good results in perturbation in ||b+Δb|| due to the ||A+ΔA||.

The reconstructed image was shown in Fig 4. The ECGLS works well in all the frequency variations. It can be produce high resolution image with maximum of 6 iterations. The resultant images of ECGLS and CGLS method, the internal visual clearness is good at 1.2 GHz frequency for ECGLS method. Both the CGLS and ECGLS method naturally have edge preserving capability. These preconditioner based regularization methods give drastic developments in the microwave tomography brain imaging.

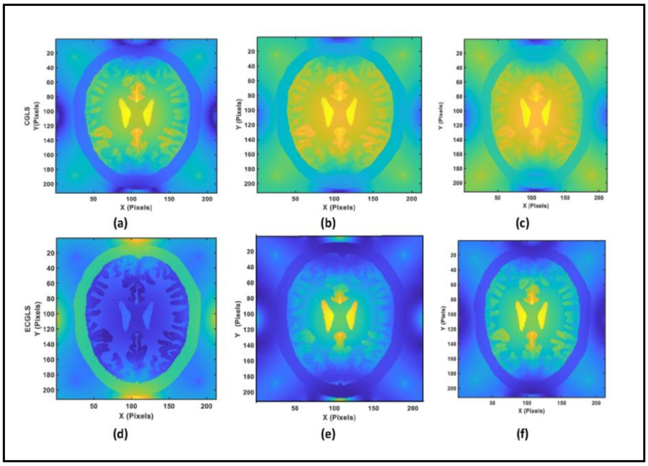

Fig. 4. Reconstructed Brain image using CGLS and ECGLS methods at 6th iteration BIM algorithm for (a, d) 0.8 GHz (b, e) 1 GHz (c, f) 1.2 GHz.

5 Conclusion

The performance of ECGLS method to solve ill-condition problem MwTIS has been analyzed. In this paper, the BIM method is implemented for the reconstruction of 2D brain phantom. The Krylov subspace regularization methods like CGLS, proposed Enriched CGLS are incorporated to analyze the numerical results of condition variations due to the frequency changes (0.8 GHz to 1.2 GHz). The stability of these methods was analyzed using relative solution error, MSE and convergence of BIM algorithm. Both the methods preserve edges in the resultant image. Compare with CGLS method, ECGLS method achieved 37.42 % of MSE at 6 BIM iterations for brain dataset.

References

[1] Wu, Z. and Wang, H. (2017). Microwave Tomography for Industrial Process Imaging. *IEEE- Antennas and Propagation magazine*, 5.

[2] Fhager, A. et al. (2018). Microwave Diagnostics Ahead. *IEEE Microwave Magazine*, 6:1527-3324.

[3] Persson, M. et al. (2014). Microwave-Based Stroke Diagnosis Making Global Pre-Hospital Thrombolytic Treatment Possible. *IEEE Trans. Biomedical Engineering*, 61(11):2806–2817.

[4] Chen, X. (2018). Computational Methods for Electromagnetic Inverse Scattering, *Wiley, IEEE Press*.

[5] Wang, Y. M. and Chew, W. C. (1989). An Iterative Solution of Two Dimensional Electromagnetic Inverse Scattering Problem. *International Journal on Imaging System Technology*, 1(1):100-108.

[6] Chew, W.C. and Wan, Y.M. (1990). Reconstruction of Two-Dimensional Permittivity Distribution Using the Distorted Born Iterative Method. *IEEE Transactions on Medical Imaging*, 9(2):218-225.

[7] Joachimowicz, N. et al. (1991). Inverse scattering: An iterative Numerical Method for Electromagnetic Imaging. *IEEE-Transaction on Antennas and Propagation*, 39(12):1742-1752.

[8] Azghani, M. (2015). Microwave Medical Imaging Based on Sparsity and an Iterative Method with Adaptive Thresholding. *IEEE Transactions on Medical Imaging*, 34(2):357-365.

[9] Azghani, M. and Marvasti, F. (2016). L2-Regularized Iterative Weighted Algorithm for Inverse Scattering, *IEEE Transactions on Antennas and Propagation*, 64(6):2293-2300.
[10] Azghani, M. (2015). Fast Microwave Medical Imaging Based on Iterative Smoothed Adaptive Thresholding. *IEEE Antennas and Wireless Propagation Letters*, 14:438-441.
[11] Miao, Z. and Kosmas, P. (2017). Multiple-Frequency DBIM-TwIST Algorithm for Microwave Breast Imaging. *IEEE Transactions on Antennas and Propagation*, 65(5):2507-2516.
[12] Desmal, A. and Bagci, H. (2014). Shrinkage-Thresholding Enhanced Born Iterative Method for Solving 2D Inverse Electromagnetic Scattering Problem. *IEEE Transactions on Antennas and Propagation*, 62(7):3878-3884.
[13] Bjork, A. et al. (1996). Numerical Methods for Least Squares Problems. *Philadelphia, PA: SIAM*.
[14] Harada, H. et al. (1995). Conjugate Gradient Method Applied to Inverse Scattering Problem. *IEEE Transaction on Antennas Propagation*, 43(8):784-792.
[15] Lobel, P. et al. (1997). Microwave Imaging: Reconstructions from Experimental Data Using Conjugate Gradient and Enhancement by Edge-Preserving Regularization. *International Journal on Imaging Systems Technology*, 8:337–342.
[16] Calvetti, D., Reichel, L. and Shuibi, A. (2002). Enriched Krylov Subspace Methods for Ill-Posed Problems. *Linear Algebra and its Applications,* 362(2003):257-273.
[17] Brain data Homepage. http://brainweb.bic.mni.mcgill.ca/brainweb/anatomic_ms.html.