The Influence of Geometric Configurations: Exploring Hexagonal and Triangular Cycles in Social Graphs

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The prioritization of substructure analysis in diverse multi-feature network studies remains an ongoing and crucial challenge. The significance of triangles is deeply rooted in engineering and architecture, exemplified by structures such as bridges and pyramids, which symbolize strength and stability. Similarly, hexagons hold considerable importance in the realm of organic chemistry. In light of these observations, we intend to investigate the pertinence of these geometric configurations and propose innovative experimental approaches to assess the strength in diverse social network graphs. By doing so, we aim to enhance the algorithm's overall credibility and contribute to the advancement of this field. This study has provided substantial evidence demonstrating that hexagonal cycles exert a significantly greater influence on the strength of social networks, with triangular cycles following closely behind.

Keywords: Feature Selection, GCN, Motifs, Network Analysis, Strength Analysis

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1 Introduction

Triangles are often regarded as one of the strongest shapes [1], although there is no conclusive scientific study that definitively proves their superiority over other shapes. Triangles are thought to be sturdy because they can uniformly disperse applied forces among their three sides. Compression is the property where force is distributed uniformly. The third side simultaneously experiences tension, which causes it to stretch. Triangles are important in graph theory as well, as shown by ideas like triadic closure and centrality measures, which centre on the development of triangular connections.

However, a different perspective contends that hexagons might possess comparable or even superior strength to triangles [2]. This presumption is supported by the observation that hexagons, like triangles, equally distribute force along all sides and possess the unique ability to seamlessly cover surfaces without overlaps. Intricate snowflake patterns and honeycombs, which have hexagonal cells that allow for optimal honey storage, serve as illustrations of these traits.

Using a hexagonal framework [3, 4] to study the kinetics of COVID transmission networks opens up a world of interesting possibilities. Hexagons have special qualities that might be useful in halting the virus's propagation. A hexagonal tessellation [3] may also be superior to conventional methods for developing vaccination distribution tactics.

Triangles concentrate on regional connectivity patterns and reveal significant nodes and characteristics within regional clusters [5]. On the contrary, hexagons offer a formal framework for recording the interactions and dependencies among global feature sets. Techniques like the Bron-Kerbosch algorithm [6], network motif [7] discovery techniques, cycle detection algorithms, and graph colouring algorithms [8] all benefit from the use of triangles and hexagons to increase their efficiency and efficacy. Given these elements, it is critical to investigate and take into account hexagons as feasible alternatives to conventional forms like triangles in order to advance engineering, scientific inquiry, and real-world applications including network analysis.

The remainder of this article is divided into the following sections. In Section 2, we give an overview of pertinent prior research that has looked at various angles on the subject of this study. Our suggested approach to solving the problem is presented in Section 3. As we move on to Section 4, we present the outcomes that show how effective our strategies were. We offer final thoughts and recommendations for future research directions in Section 5.

2 Related Work

Feature selection plays a vital role in Graph Neural Networks (GNNs), warranting extensive research efforts to identify the most critical features that contribute to model performance. Notably, Mahmoud et al. [9] leveraged centrality measures to select influential features, demonstrating promising outcomes when integrated into their proposed model. Similarly, Thoma et al. [10] introduced a novel approach for feature selection based on frequent subgraphs, which hold significance in graph classification tasks owing to their prevalence and discriminated capabilities.

Several research efforts have been dedicated to improving node classification in graph analysis. Rong et al. [11] introduced a novel approach known as DropEdge to mitigate the limitations observed in Graph Convolutional Networks (GCNs). Chakrabarti et al. [12] leveraged link information to enhance the accuracy of their model. Wang et al. [13] proposed an extension to the basic GCN model called MSF-GCN, which incorporated attribute similarity along with network topology considerations. In our study, we strive to enhance the accuracy of our Graph Convolutional Network (GCN) model by integrating features associated with the triangular and hexagonal cycles present in the graph dataset.

The current limitations in accuracy of graph classification tasks using Graph Convolutional Networks (GCNs) have motivated the exploration of novel approaches. Nagar et al. [14] introduced Quadratic GCN (QGCN), a pioneering concept that outperformed classical GCN models and yielded significantly improved results. Another notable contribution by Bian et al. [15] involved the development of a bidirectional GCN tailored for rumor detection in social networks, effectively leveraging both top-down and bottom-up graph structures. Wilkens et al. [16] pursued accuracy enhancement through an ensemble of GCNs. Furthermore, alternative models have been employed to tackle graph classification tasks. For example, Lee et al. [17] proposed an innovative Recurrent Neural Network (RNN) model incorporating attention mechanisms to selectively focus on the most informative portion of the graph.

Motifs have been extensively utilized across various domains, including biological networks [18] and network traffic analysis [19]. In the realm of network traffic analysis [19], motifs play a crucial role in predicting the type of applications present within the network. By identifying significant motifs, a node's description is formulated based on its involvement in these motifs. On the other hand, in the study of metabolic networks [17], a distinct motif concept termed ”reaction motifs” is employed to analyze and extract valuable insights from metabolic networks. Researchers have also introduced innovative techniques for the extraction of motifs from graphs [20] owing to their influential role in modeling frameworks.

3 Methodology

In the methodology of a strength analysis algorithm is presented, incorporating Graph Energy and Algebraic Connectivity with a focus on triangular and hexagonal cycles. Further node and graph classification experiments are discussed to evaluate the effectiveness of these features in improving model accuracy. Motif analysis provides insights into the occurrence of specific patterns within the graphs.

3.1 Strength Analysis

In the context of substructure prioritization, we present an advanced strength analysis algorithm that incorporates the computation of Graph Energy [?] and Algebraic Connectivity while harnessing the inherent graph structure, particularly the triangular and hexagonal cycles. Graph Energy, a significant metric, entails the summation of the squared absolute values of the eigenvalues derived from the adjacency matrix of the graph. Additionally, Algebraic Connectivity refers to the second smallest eigenvalue obtained from the Laplacian matrix, which encodes the interconnections between the graph's nodes.

We employ the first Harry Potter novel “Harry Potter and the Philosopher's Stone” to simulate an undirected social graph of characters and relationships. The algorithm necessitates the calculation of cycles of length 3 and 6, described in Algorithm 1 and Algorithm 2, respectively. Initially, the algorithm computes the graph energy of all cycles with a length of 3. Subsequently, a node is randomly removed [24] (representing cycles of length 2), and the resulting graph energy is computed. The algorithm then determines the graph energy of cycles with a length of 6, followed by the arbitrary removal of a node [24] (corresponding to cycles of length 5) and subsequent graph energy calculation. A similar approach is adopted for assessing Algebraic Connectivity.

In a real-world social graph, where edges are highly random, the increment in the number of nodes does not necessarily lead to an increase in graph energy and algebraic connectivity. Our experimental results demonstrate a substantial disparity in the metrics, indicating significantly higher mean graph energy and algebraic connectivity in cycles of lengths 3 and 6. Notably, the strength metrics reveal that hexagonal cycles exhibit greater strength compared to triangular cycles.

3.2 Node classification

In order to gain deeper insights into the significance of triangular and hexagonal cycles, we performed node classification on the Zachary's Karate Club social network dataset. This task revolves around the detection of two communities of individuals who aligned with either the administrator or the instructor when a conflict arose between these two influential figures. Our initial approach, referred to as Model 1, employed a baseline model which utilized the identity matrix as the feature matrix for the Graph Convolutional Network (GCN) model [25]. This model adopted a two-layer architecture with hidden layers and leveraged the use of the spectral rule [26]. For node classification [27], a logistic regression layer was incorporated. After careful experimentation, the optimal number of epochs chosen for the model was 100. To improve the accuracy of our model, we introduced additional features based on the number of triangular and hexagonal cycles associated with each node. The computation of these cycles was performed using Algorithm 1 and Algorithm 2, which formed the foundation of Model 2. Furthermore, we developed two additional models: Model 3 focused solely on triangular cycles, while Model 4 exclusively considered hexagonal cycles. All models incorporated the identity matrix as a feature, which was concatenated with supplementary features in the case of Models 2, 3, and 4. The utilization of these additional features resulted in notable accuracy improvements across all models. Subsequent experimentation involved modifications to the utilized features. For instance, we introduced a binary system wherein nodes were assigned a value of 1 if they belonged to at least one cycle, and 0 if they did not. However, these modified models yielded insignificant improvements as compared to the mentioned models. This lack of significant results can be attributed to the relatively small scale of the network, consisting of only 34 nodes, with a significant majority of nodes being part of a cycle, given the social nature of the graph.

3.3 Graph Classification

The scope was expanded to include a distinct category of non-social dataset termed the WICO Graph Dataset. This dataset posed a Graph Classification Problem [28], which deviated from the previously explored node classification paradigm [27]. This extension aimed to explore the efficacy of the proposed methodology in tackling the challenging task of graph classification in a context characterized by conspiratorial networks.

This assessment is performed using Algorithm 1, which allows for the determination of the triad count as an additional node-centric feature. The resulting augmented graph, denoted as G_t, encapsulates the enriched network representation derived from the triad analysis. To further probe the graph's intricate structural characteristics, a similar procedure is employed to identify hexagons within the network. Algorithm 2 is utilized to calculate the hexagon count for each node, capturing the extent of node involvement in such topological formations. Consequently, the resulting graph, G_h, represents an expanded and refined network representation, empowered by the integration of hexagon analysis. These meticulous procedures enable a comprehensive understanding of the underlying network structure, facilitating the extraction of valuable insights and the identification of discernible patterns pertaining to node participation in triads and hexagons.

We further employed tensorized representations of Graphs G_t and G_h as input data to train a Graph Convolutional Network (GCN) [29, 30] classification model aimed at distinguishing Conspiracy graphs from Non-Conspiracy Graphs. The dataset comprises 816 graphs, which are segregated into input graphs and their associated target labels. Diverse combinations of train-test splits are systematically investigated to gauge the model's performance and optimize classification accuracy. Multiple models are developed, each focusing on the exploration of diverse feature sets for graph classification. **Model 1** incorporates the number of friends and followers as input features for classification. **Model 2** expands upon Model 1 by including an additional feature, the count of triangles in which each node participates. **Model 3** goes a step further by incorporating the count of hexagons as an additional feature alongside friends and followers. **Model 4** combines all the aforementioned features, including friends, followers, count of triangles, and count of hexagons, to create a comprehensive input representation for classification. The architecture is represented in Figure 1.

3.4 Motif Analysis

Motifs represent recurring patterns or subgraphs within a network that hold significance across various applications. These motifs capture specific configurations of nodes and edges, such as feed-forward loops, cliques, and chain-like structures, conveying unique insights into network properties.

Notably, hexagonal cycles did not emerge as motifs within the dataset. Although hexagonal cycles exist in the graphs, they are limited to only a few nodes among many, rendering them ineligible as substructure patterns and consequently

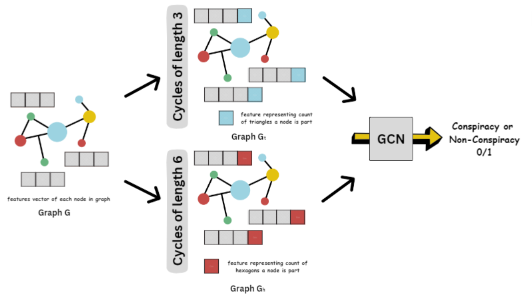

Figure 1: Proposed architecture for Graph classification

absent from the motif analysis. To further investigate, we conducted a comparative analysis between triangular cycles in the WICO Graph Dataset and the previously mentioned Harry Potter dataset. Considering the diverse range of graph sizes in the WICO Graph Dataset, we specifically focused on graphs with 25 to 30 nodes to align with the 28 nodes in the Harry Potter dataset. By examining the values in the last column of the identified motifs, i.e., triangular cycles, we found that the Harry Potter dataset exhibited 285 cycles, whereas the WICO Graph Dataset averaged only 54 cycles. Unlike explicit social graphs, conversational graphs are implicit social graphs and may not necessarily exhibit large conversational patterns like hexagons.

4 Results

4.1 Strength Analysis

Notably, the strength analysis metrics reveals that hexagonal cycles exhibit greater strength compared to triangular cycles. A detailed presentation of these findings on Harry Potter dataset is provided in Table 1.

Table 1: Results of the strength analysis for Harry Potter dataset

Cycle Length	Mean Graph Energy	Mean Algebraic Connectivity
2 (connection between nodes)	8865.89	-1.07
3	20329.43	20.36
5	2456.00	26.06
6	79143.89	122.45

4.2 Node Classification

The accuracies attained by the four models for node classification are presented in Table 2. Furthermore, to illustrate the progression of the loss function during the training process, we have plotted the variation of loss against epoch in Figure 2. A higher rate of error decrease in the case of feature vectors with hexagon and triangular count can indeed be indicative of better progress, which contributes to determining the overall performance and superiority of a model. The higher rate of error decrease suggests that the model incorporating these feature vectors is more effectively leveraging the geometric properties of hexagons and triangles to make accurate predictions.

Comparative analysis revealed that Model 4 outperformed Model 3, albeit falling short of the accuracy achieved by Model 2. This observation aligns with the findings obtained from strength analysis, suggesting that the inclusion of hexagonal cycles significantly improves model accuracy. Consequently, it can be inferred that both triangular and hexagonal cycles play crucial roles in increasing the accuracy of the model.

Table 2: Results for Node classification of Karate Club dataset

Type of analysis/Model	Model 1	Model 2	Model 3	Model 4
Node classification	0.50	0.53	0.66	0.72

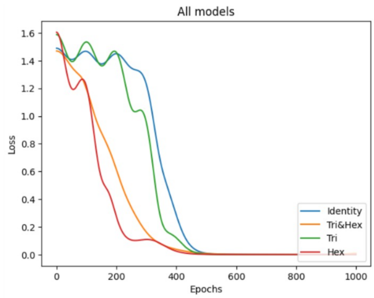

Figure 2: Loss vs Epochs for different models

4.3 Graph Classification

In this case, the accuracy of the predictions for each of these baselines turned out to be nearly the same. Although adding the count of triangles and hexagons did not significantly improve performance, it also did not have a detrimental effect. The results across all these baselines remained consistent.

Several factors may have contributed to the observed results. Firstly, the effectiveness of incorporating the count of triangles and hexagons as features was evident in ”social graphs” such as the Harry Potter and Karate Club datasets, which naturally exhibited a high occurrence of triangular and hexagonal cycles. However, this characteristic might not be as prevalent in the WICO Graph Dataset, potentially impacting the results. Secondly, a notable discrepancy was observed in the presence of hexagonal cycles within the WICO Graph Dataset. Approximately 50% of the graphs in the dataset contained hexagonal cycles, while the remaining graphs did not. This imbalance in cycle presence could have influenced the accuracy outcomes.

Table 3: Results for Graph classification for WICO Graph Dataset

Type of analysis/Model	Model 1	Model 2	Model 3	Model 4
Graph classification	0.50	0.56	0.44	0.56

Overall the findings presented in this research highlights the significance of considering subgraph or cycle characteristics in the analysis of graph datasets. The observed disparities as well as the impact of additional features, emphasize the importance of feature selection and its influence on accuracy improvements.

5 Conclusion and Future Scope

In this article, we present an investigation that offers a novel method and viewpoint for choosing and prioritising features when developing network algorithms and training graph models. We suggest a special training algorithm designed to incorporate hexagonal and triangular features, with the goal of solving the issue at hand. Our main goal is to demonstrate the importance of these structural characteristics, supported by empirical data from natural phenomena, in order to move beyond theoretical hypotheses. Additionally, in order to advance the understanding of feature selection and graph structure learning for the purpose of social network analysis, our work aims to advance this area of study. We confidently claim, based on our thorough analysis, that hexagonal and triangular cycles have a significant impact on predicting social graph outputs. We observed that hexagonal cycles have a greater impact and consequently, produced a higher accuracy as compared to that of triangular cycles. Four distinct tasks—Strength analysis, Node classification, Graph classification, and Motif analysis—are carried out to support this claim.

In terms of future directions, we pinpoint two essential elements for enhancing this study. Firstly, there are efficiency issues because the current method for identifying hexagonal cycles involves a computational complexity of $O(N^6)$, resulting in significant computation time. To address this, a timeout constraint of 5 minutes was implemented. However, this constraint may have affected the overall performance of the models. We suggest looking into potential strategies to reduce or optimise this complexity in order to speed up and improve the viability of the identification process. Second, we would like to expand this work to a variety of other domains and broaden its application beyond social networks. We believe that by addressing these issues, the suggested methodology will become more useful and applicable while also advancing our understanding of the subject. Thirdly, we plan to explore this on a large explicit social graph dataset where edges are based on explicit relations.

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1 Data

The text version of the novel "Harry Potter and the Sorcerer's Stone" [1] has also been utilized as one of the datasets.

Zachary's karate club dataset [2] represents a social network of a university karate club. This well-known dataset consists of 78 edges and is publicly available. The data is represented as a list of integer pairs, where each integer represents a member of the karate club, and a pair indicates an interaction between two members.

The WICO Graph Dataset[3] consists of a comprehensive collection of around 400 unique instances of Conspiracy and Non-Conspiracy Graphs. These instances are organized into two folders and are accompanied by two important files: "node.csv" and "edges.txt". The "edges.txt" file provides detailed information about the connections within the undirected graph, specifying the starting and ending nodes for each edge. On the other hand, the "node.csv" file contains specific data on the number of friends and followers associated with each distinct node ID, offering insights into the network dynamics. An instance of the dataset is shown in Figure 1.

2 Experimental Environment

Python 3.10.11 [4] was used to implement all the algorithms, and GoogleColab[5] was used to implement all the tests. The deep learning framework that we used was PyTorch[6]. We intend to fully publish the research and dataset on GitHub[7] for the benefit of the research community.

3 Algorithm Description

[1] https://github.com/pprzetacznik/nlp-n-grams/blob/master/train_corpus/Harry%20Potter%201%20Sorcerer%27s_Stone.txt

[2] https://networkx.org/documentation/stable/auto_examples/graph/plot_karate_club.html/

[3] https://datasets.simula.no/wico-graph/

[4] https://www.python.org/downloads/release/python-31011/

[5] https://colab.research.google.com/

[6] https://pytorch-geometric.readthedocs.io/en/latest/

[7] https://github.com/Chaitra-Bhat383/HexagonsV-STriangles

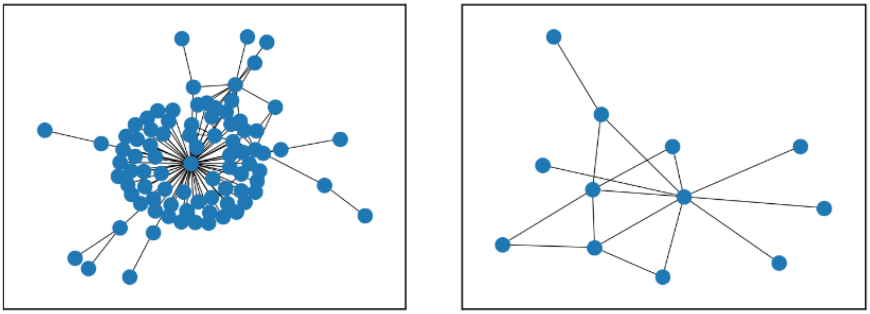

Figure 3: The Figure on the left represents a Non-Conspiracy Graph and Figure on the right represents a Conspiracy Graph

Algorithm 2 Algorithm to generate triangular cycles

$cycles \leftarrow []$ $i \in N$ $j \in N$ $edge[i,j] \in E$ $k \in V$ $i \neq k$ && $edge[i,k] \in E$ && $edge[k,j] \in E$ $[i,j,k] \notin cycle$ && $[j,i,k] \notin cycle$&&$[k,i,j] \notin cycle$&&$[i,k,j] \notin cycle$&&$[j,k,i] \notin cycle$&&$[k,j,i] \notin cycle$ $cycle = cycle + [i,j,k]$

Algorithm 3 Algorithm to generate hexagonal cycles

$cycles \leftarrow []$ $temp \leftarrow []$ $i \in N$ $j \in N$ $edge[i,j] \in E$ $k \in V$ $i \neq k$ && $edge[k,j] \in E$ $l \in V$ $i \neq l$&&$j \neq l$ && $edge[l,k] \in E$ $m \in V$ $i \neq m$&&$j \neq m$ && $k \neq m$ && $edge[m,l] \in E$ $n \in V$ $i \neq n$&&$j \neq n$ && $k \neq n$ && $l \neq n$ && $edge[n,m] \in E$ $temp \leftarrow [i,j,k,l,m,n]$ $temp \notin cycles$ $cycles = cycles + temp$