$M^{[X]}/G^{(1,K)}/1$queue with Unreliable Server and with Compulsory Vacation

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A batch arrival and batch service queue have been considered. The arrival pattern is Poisson. After completion of each service, the server takes a compulsory vacation, and the server may also break down at random time epochs during service. For this queuing model, using the supplementary variable technique, the probability-generating function of the number of customers in the queue at different server states has been obtained. The mean queue size and the probability of the server being idle have been derived. Some particular models have been derived. Some numerical models have been derived.

Keywords: queuing, vacation, unreliable server, probability generating function, and operating characteristics.

AMS Subject classification number: 90B22, 60K25 and 60K30.

1 Introduction

The study of mathematical modeling of queueing system plays a vital role in day-to-day life as congestion situations occur in almost all the fields of research. For example, many people access particular telephone towers for getting service at the same time, similarly, many persons using internet felicity may search a particular website at the same time, etc. Usually after completion of service to the customers who are currently in the service station, the server may take rest or go for a small break or go for maintenance. The duration of this period is called vacation period of the server.

2023. In Deepak Bhatia & Himanshu Arora (eds.), *SCRS Proceedings of International Conference of Undergraduate Students 2023*, 107–117. Computing & Intelligent Systems, SCRS, India. https://doi.org/10.56155/978-81-955020-4-2-11

The researcher incorporated vacation concept in queuing models as it best suit with real-life problems. Doshi (1986) and Takagi (1991) have done excellent survey works on vacation queues. Maragathasundari S et al. (2018) analyzed a single server general service queue with compulsory vacation. In today's world, the concept of unreliable servers is very common in any type of service. So the researchers added the unreliable state of the server in queuing models. In particular, the vacation queueing models studied with the concept of unreliable servers. some related research works are: Li et al (1997) studied reliability analysis of a general service queue with vacation and server breakdown, Grey et al(2000) analyzed a vacation queue with server breakdown, Ke (2005) considered Modified T Vacation policy queue with the unreliable server, Wang et al (2005) studied comparative analysis for the N-policy general service queue with removable unreliable server. Vanitha (2019) studied a single server queue with general service time, compulsory vacation, and three-phase repairs. Vanitha (2020) analyzed a M/D/1 queue with compulsory vacation and random breakdowns.

Sometimess the server provides different types of service's to arriving customers. Kalyanaraman and Suvitha (2018) considered a single server queue with compulsory vacation with two types of services and with restricted admissibility.

The study on batch arrival batch service queue with some other assumptions is carried out by some authors. Madan et al(2003) considered the steady state analysis of batch arrival batch service queue with breakdown. A Batch arrival queue with variant vacation policy and balking was studied by Ke (2007). The batch arrival queue with randomized vacation and the unreliable server was analyzed by Ke et al (2012). Uma and Punniyamoorhthy (2016) studied bulk queues with feedback, two choice of service and compulsory vacation. Kalyanaraman and Nagarajan (2016) considered batch arrival fixed batch service queue with unreliable servers and with vacation. Uma and Manikandan (2017) analyzed bulk queueing system with three stages of heterogeneous service, compulsory vacation and balking.

In this article, we consider an $M^{[X]}/G^{(1,K)}/1$ queue with an unreliable server and with compulsory server vacation. This type of queuing system exists in manufacturing industries, Transportation system etc. In manufacturing industries, after products are approved for transportation to customer shops, they are transported to the shops in bulks by trucks. After transporting the products, the truck will be used for other work or the truck is sent for maintenance (vacation period). During the service period (transportation period), the truck may breakdown. The above situation can be modeled as an $M^{[X]}/G^{(1,K)}/1$ queue with unreliable server and compulsory server vacation.

The remainder of the article is organized as follows: Section 2 provides the model description and mathematical analysis. In section 3, we obtain the mean queue size of the model discussed in this paper. In section 4, we present some particular models. In section 5, we present numerical results. In section 6, we present a conclusion.

2 The Model and Analysis

The arrival process of customers follows the Poisson process with parameter λ>0, the customer arrives in batches of variable size X, where X is a random variable with probability P{X=j}=C_j, whose probability generating function is defined by $C(z) = \sum_{j=1}^{\infty} C_j z^j$.

The server provides service to customers in batches of variable size with minimum batch size 1 and maximum batch size $K(> 0)$. The service time distribution of each batch is a random variable and it is assumed as generally distributed with distribution function $G(x)$. After completion of service to each batch of customers, the server takes a compulsory vacation of random duration independent of a number of customers in the queue. The duration of the vacation period is a random variable and it is assumed as generally distributed with distribution function $B(x)$.

In addition the server may break down during a service to a batch of customers, the breakdowns are assumed to occur according to a Poisson process with rate 'α'. Once the server breaks down, the batch whose service is interrupted goes to the head of the queue and the repair to server starts immediately. The duration of the repair period is generally distributed with the distribution function $H(x)$.

The analysis of this model is based on the supplementary variable technique and the supplementary variable is elapsed service time / elapsed vacation time / elapsed repair time.

For analysis we define the conditional probabilities and probability as follows:

$\mu(x) = \frac{g(x)}{1-G(x)}$is the conditional probability that the completion of service during the interval $(x, x + dx]$, given that the elapsed service time is 'x'.

$\beta(x) = \frac{b(x)}{1-B(x)}$is the conditional probability that the completion of vacation during the interval $(x, x + dx]$, given that the elapsed vacation time is x'.

$\gamma(x) = \frac{h(x)}{1-H(x)}$is the conditional probability that the completion of repair during the interval $(x, x + dx]$, given that the elapsed repair time is 'x'.

The Markov process related to this model is$\{(N(t), S(t)) : t \geq 0\}$ where $N(t)$ be the number of customer in the queue and $S(t)$ be the supplementary variable at time t. and is defined as

$$\begin{aligned} S(t) &= S_1(t), \text{ elapsed service time} \\ &= S_2(t), \text{ elapsed vacation time} \\ &= S_3(t), \text{ elapsed repair time} \end{aligned}$$

$P_n(t, x)$=P{At time 't', there are 'n' customers in the queue(excluding the customer in service) and the elapsed service time is 'x'}

$V_n(t, x)$=P{At time 't', there are 'n' customers in the queue and the elapsed vacation time is 'x'}

$R_n(t, x)$=P{At time 't', there are 'n' customers in the queue and the elapsed repair time is 'x'}

and

$Q(t)$=P{At time 't', there are no customer in the queue and the server is idle}

The differential − difference equations for this model are

$$\frac{dP_0(x)}{dx} = -(\lambda + \mu(x) + \alpha)P_0(x) \tag{11.1}$$

$$\frac{dP_n(x)}{dx} = -(\lambda + \mu(x) + \alpha)P_n(x) + \lambda \sum_{j=1}^{n} C_j P_{n-j}(x), \quad \text{for } n \geq 1 \tag{11.2}$$

$$\frac{dV_0(x)}{dx} = -(\lambda + \beta(x))V_0(x) \tag{11.3}$$

$$\frac{dV_n(x)}{dx} = -(\lambda + \beta(x))V_n(x) + \lambda \sum_{j=1}^{n} C_j V_{n-j}(x), \quad \text{for } n \geq 1 \tag{11.4}$$

$$\frac{dR_0(x)}{dx} = -(\lambda + \gamma(x))R_0(x) \tag{11.5}$$

$$\frac{dR_n(x)}{dx} = -(\lambda + \gamma(x))R_n(x) + \lambda \sum_{j=1}^{n} C_j R_{n-j}(x), \quad \text{for } n \geq 1 \tag{11.6}$$

$$0 = -\lambda Q + \int_0^\infty R_0(x)\gamma(x)dx + \int_0^\infty V_0(x)\beta(x)dx \tag{11.7}$$

The boundary conditions are

$$P_0(0) = \int_0^\infty \sum_{j=1}^{K} V_j(x)\beta(x)dx + \int_0^\infty \sum_{j=1}^{K} R_j(x)\gamma(x)dx + \lambda Q \tag{11.8}$$

$$P_n(0) = \int_0^\infty V_{n+K}(x)\beta(x)dx + \int_0^\infty R_{n+K}(x)\gamma(x)dx \quad \text{for} \quad n \geq 1 \tag{11.9}$$

$$V_n(0) = \int_0^\infty P_n(x)\mu(x)dx, \text{ for } n \geq 0 \tag{11.10}$$

$$R_n(0) = \alpha \int_0^\infty P_{n-1}(x)dx, \text{ for } n \geq 1 \tag{11.11}$$

$$R_0(0) = 0 \tag{11.12}$$

and the normalization condition is

$$Q + \int_0^\infty \sum_{n=0}^{\infty} [P_n(x) + V_n(x) + R_n(x)]dx = 1 \tag{11.13}$$

For the analysis, we define the probability generating functions as follows:
$P(x,z) = \sum_{n=0}^{\infty} P_n(x)z^n$, $V(x,z) = \sum_{n=0}^{\infty} V_n(x)z^n$, $R(x,z) = \sum_{n=0}^{\infty} R_n(x)z^n$,

$C(z) = \sum_{j=0}^{\infty} C_j z^j$,
where
Let

$$S(z) = P(z) + V(z) + R(z), \tag{11.14}$$

S(z) represents the probability generating function of number of customer in the queue, independent of server state.

$$S(z) = \frac{\{[m + \alpha z(1 - H^*(m)](1 - G^*(a)) + aG^*(a)[1 - B^*(m)]\}A}{m\{az^K - aB^*(m)G^*(a) - \alpha z[1 - G^*(a)]H^*(m)\}} \tag{11.15}$$

Let S(z)=$\frac{I_1 A}{m I_2}$, where
$I_1 = [m + \alpha z(1 - H^*(m))][1 - G^*(a)] + aG^*(a)[1 - B^*(m)]$
$I_2 = az^K - aB^*(m)G^*(a) - \alpha z[1 - G^*(a)]H^*(m)$
Since $Q + S(1) = 1$, which implies,
$Q = 1 - S(1)$

S(1)=0/0, So apply L'hospital rule two times, we have

S(1)=$\frac{I_1' A'}{m' I'_2}{}_{/z=1}$, where

$I'_{1/z=1} = -\lambda E(X)[1 - G^*(a)][1 + \alpha E(R)] + \alpha G^*(\alpha)E(V)$

$I'_{2/z=1} = \alpha K - \lambda E(X)[1 - G^*(\alpha)] - \alpha\lambda E(x)E(V)G^*(\alpha)$
$- \alpha[1 - G^*(\alpha)][1 + \lambda E(R)E(X)]$

$A'_{/z=1} = K\lambda Q$
$+ \int_0^\infty \sum_{n=1}^K (K - n)V_n(x)\beta(x)dx + \int_0^\infty \sum_{n=1}^K (K - n)R_n(x)\gamma(x)dx$

3 Mean Queue Size

In this section, we calculate the mean queue size.

Let us differentiate S(z) with respect to z and by substituting z=1, we get the mean queue size.

After differentiating S(z) in (25) with respect to z, we have

S'(z)=$\frac{mI_2[I_1' A+I_1 A']-I_1 A[m' I_2+I_2' m]}{[mI_2]^2}$

Substituting z=1, we have S'(1)=0/0,

After applying L'hospital rule four times, we have

S'(1)=$\frac{2\{m' I_2[I_1' A''+I_1'' A']-I_1' A'[m'' I_2'+I_2'' m']\}}{6[m' I_2']^2}{}_{/z=1}$

$I''_{1/z=1} = -2\lambda^2 E^2(X)[1 + \alpha E(R)][G^{*'}(\alpha)]+ [1 - G^*(\alpha)][-\lambda E(X(X - 1))$
$-2\alpha\lambda E(R)E(X) - \alpha(\lambda^2 E^2(X)E(R^2) + \lambda EX(X-1)E(R))] + 2\lambda^2 E^2(X)G^*(\alpha)E(V)$
$+ 2\alpha\lambda^2 E^2(X)G^{*'}(\alpha)E(V) - \alpha\lambda EX(X-1)G^*(\alpha)E(V) - \alpha\lambda^2 E^2(X)G^*(\alpha)E(V^2)$

$I''_{2/z=1} = \alpha K(K - 1) - 2\lambda K E(X) - \lambda EX(X - 1) + \lambda EX(X - 1)G^*(\alpha)$
$+2\lambda^2 E^2(X)E(V)G^*(\alpha) - 2\lambda^2 E^2(X)G^{*'}(\alpha) + 2\alpha\lambda^2 E^2(X)E(V)G^{*'}(\alpha)$
$-\alpha[\lambda^2 E^2(X)E(V^2) + \lambda E(V)EX(X - 1)]G^*(\alpha) - 2\alpha\lambda E(X)G^{*'}(\alpha)$
$- 2\alpha\lambda E(X)[1 - G^*(\alpha)]E(R) - 2\alpha\lambda^2 G^{*'} E(R)E^2(X)$
$-\alpha[1 - G^*(\alpha)][\lambda^2 E^2(X)E(R^2) + \lambda EX(X - 1)E(R)]$

$A''_{/z=1} = \int_0^\infty \sum_{n=1}^K (K(K - 1) - n(n - 1))V_n(x)\beta(x)dx$
$+ \int_0^\infty \sum_{n=1}^K (K(K - 1) - n(n - 1))R_n(x)\gamma(x)dx + (K(K - 1))\lambda Q$

4 Some Particular Models

In this section, some particular models are derived related to the model discussed in this article by replacing the general distribution by known.

Case(i) $M^{[X]}/G^{(1,K)}/1$ queue with break down and with compulsory vacation

Here we take, $G(x) = 1 - e^{-\mu x}$; $B(x) = 1 - e^{-\beta x}$; $H(x) = 1 - e^{-\gamma x}$ and assume that batch arrival size random variable X follows geometric distribution with probability $C_n = (1-s)^{n-1}s$ for $n \geq 1$, where s=1-t.

The probability generating function of number of customers in the queue, independent of the server state.

$$S(z) = \frac{\{[m(\gamma+m) + \alpha z m]a(\beta+m) + a\mu m(\gamma+m)\}A}{m\{az^K(\mu+a)(\beta+m)(\gamma+m) - a\beta\mu(\gamma+m) - \alpha z a\gamma(\beta+m)\}} \tag{11.16}$$

where

$$A = \int_0^\infty \sum_{n=1}^{K}(z^K - z^n)V_n(x)\beta(x)dx + \int_0^\infty \sum_{n=1}^{K}(z^K - z^n)R_n(x)\gamma(x)dx + (z^K - 1)\lambda Q$$

m=λ -λ C(z), a=λ -λ C(z)+α

The mean queue size

S'(1)=$\frac{2\{m' I_2[I_1' A'' + I_1'' A'] - I_1' A'[m'' I_2' + I_2'' m']\}}{6[m' I_2']^2}_{/z=1}$, where

$I'_{1/z=1} = -\frac{\lambda}{s\beta\gamma(\mu+\alpha)}[\alpha(\gamma+\alpha)\beta + \alpha\mu\gamma]$

$I'_{2/z=1} = \frac{\alpha K s\beta\gamma(\mu+\alpha) - \lambda\alpha\beta\gamma - \alpha\lambda\mu\gamma - \alpha^2(\gamma s+\lambda)\beta}{s\beta\gamma(\mu+\alpha)}$

$A'_{/z=1} = \int_0^\infty \sum_{n=1}^{K}(Kz^{K-1} - nz^{n-1})V_n(x)\beta(x)dx$
$+ \int_0^\infty \sum_{n=1}^{K}(Kz^{K-1} - nz^{n-1})R_n(x)\gamma(x)dx + (Kz^{K-1})\lambda Q$

$I''_{1/z=1} = \{2\lambda^2\mu(\alpha+\gamma)\beta^2\gamma - \alpha\beta^2(\mu+\alpha)[2\lambda(1-s)\gamma^2 + 2\alpha K\lambda s\gamma + 2\alpha\lambda\gamma(1-s)$
$+2\lambda^2\alpha] + 2\lambda^2\mu(\mu+\gamma)\beta\gamma^2 - 2\alpha\lambda^2\mu\beta\gamma^2 - 2\alpha\lambda(1-s)\mu(\mu+\alpha)\beta\gamma^2$
$- 2\alpha\lambda^2\mu(\mu+\alpha)\gamma^2\}\{\frac{1}{s^2\beta^2\gamma^2(\mu+\alpha)^2}\}$

$I''_{2/z=1} = \{\alpha K(K-1)s^2\beta^2\gamma^2(\mu+\alpha)^2 + \beta^2\gamma^2(\mu+\alpha)[-2\lambda K s(\mu+\alpha)$
$- 2\lambda(1-s)(\mu+\alpha) + 2\lambda(1-s)\mu] + \beta\gamma^2[2\lambda^2\mu(\mu+\alpha) + 2\lambda^2\mu\beta - 2\alpha\lambda^2\mu]$
$+\gamma^2[-\alpha\mu(2\lambda^2 + 2\lambda(1-s)\beta)(\mu+\alpha) + 2\alpha\lambda\mu\beta^2 s] - 2\alpha^2\beta^2\lambda s\gamma(\mu+\alpha)$
$+2\alpha\lambda^2\beta^2\mu\gamma - \alpha^2\beta^2[2\lambda^2 + 2\lambda(1-s)\gamma](\mu+\gamma)\}\{\frac{1}{s^2\beta^2\gamma^2(\mu+\alpha)^2}\}$

$A''_{/z=1} = \int_0^\infty \sum_{n=1}^{K}(K(K-1) - n(n-1))V_n(x)\beta(x)dx$
$+ \int_0^\infty \sum_{n=1}^{K}(K(K-1) - n(n-1))R_n(x)\gamma(x)dx + K(K-1)\lambda Q$

Case(ii) $M^{[X]}/G/1$ queue with breakdown and with compulsory vacation

Here we take K=1,

The probability generating function corresponding to number of customers in the queue, irrespective of the state of the server.

$$S(z) = \frac{\{[m+\alpha z(1-H^*(m)](1-G^*(a))+aG^*(a)[1-B^*(m)]\}A}{m\{az-aB^*(m)G^*(a)-\alpha z[1-G^*(a)]H^*(m)\}}$$

where $A = (z-1)\lambda Q$, m=λ -λ C(z), a=λ -λ C(z)+α

The mean queue size

$$S'(1)=\frac{2\{m' I_2[I_1' A''+I_1'' A']-I_1' A'[m'' I_2'+I_2'' m']\}}{6[m' I_2']^2}\Big/_{z=1}$$

$$I'_{1/z=1} = -\lambda E(X)\{[1-G^*(a)][1+\alpha E(R)]+\alpha G^*(\alpha)E(V)\}$$

$$I'_{2/z=1} = \alpha-\lambda E(X)[1-G^*(\alpha)]-\alpha\lambda E(x)E(V)G^*(\alpha)-\alpha[1-G^*(\alpha)][1+\lambda E(R)E(X)]$$

$$A'_{/z=1} = \lambda Q$$

$$\begin{aligned} I''_{1/z=1} &= -2\lambda^2 E^2(X)[1+\alpha E(R)][G^{*'}(\alpha)] + [1-G^*(\alpha)][-\lambda E(X(X-1)) \\ &\quad -2\alpha\lambda E(R)E(X) - \alpha(\lambda^2 E^2(X)E(R^2)+\lambda EX(X-1)E(R))] \\ &\quad +2\lambda^2 E^2(X)G^*(\alpha)E(V) \\ &\quad +2\alpha\lambda^2 E^2(X)G^{*'}(\alpha)E(V) - \alpha\lambda EX(X-1)G^*(\alpha)E(V) - \alpha\lambda^2 E^2(X)G^*(\alpha)E(V^2) \end{aligned}$$

$$\begin{aligned} I''_{2/z=1} &= -2\lambda E(X) - \lambda EX(X-1) + \lambda EX(X-1)G^*(\alpha) \\ &\quad +2\lambda^2 E^2(X)E(V)G^*(\alpha) - 2\lambda^2 E^2(X)G^{*'}(\alpha) + 2\alpha\lambda^2 E^2(X)E(V)G^{*'}(\alpha) \\ &\quad -\alpha[\lambda^2 E^2(X)E(V^2)+\lambda E(V)EX(X-1)]G^*(\alpha) - 2\alpha\lambda E(X)G^{*'}(\alpha) \\ &\quad -2\alpha\lambda E(X)[1-G^*(\alpha)]E(R) - 2\alpha\lambda^2 G^{*'}(\alpha)E(R)E^2(X) \\ &\quad -\alpha[1-G^*(\alpha)][\lambda^2 E^2(X)E(R^2)+\lambda EX(X-1)E(R)] \end{aligned}$$

$$A''_{/z=1} = 0$$

5 Numerical results

In this section, we present some numerical example related to the model I in section 4. We fix the values of the parameters $\mu, \alpha, \gamma, \beta, s$ and we vary the values of arrival rate λ. For various values of K, we find the values of E(N). Also we find the values of Q. The results are presented in tables 1 to 2. From the values, it is clear that, as the arrival rate increases, the probability of server being idle decreases, also the mean number of customers in the queue increases, for increasing values of arrival rate. Which is very much coincide with our expectation.

Table 1: Probability of Server Being Idle and Mean Queue Size
($\alpha = 10, \beta = 15, \mu = 25, \gamma = 15, s = 0.9$)

λ	Probability of server being idle					Mean Queue Size				
	K=2	K=3	K=4	K=5	K=6	K=2	K=3	K=4	K=5	k=6
1	0.8045	0.7846	0.7754	0.7701	0.7668	0.0596	0.0674	0.0707	0.0725	0.0736
2	0.7102	0.6829	0.6699	0.6625	0.6577	0.1163	0.1277	0.1333	0.1364	0.1383
3	0.6219	0.5901	0.5736	0.5641	0.5579	0.1767	0.1873	0.1946	0.1991	0.2019
4	0.5398	0.5066	0.4871	0.4751	0.4674	0.2435	0.2466	0.2549	0.2610	0.2652
5	0.4640	0.4330	0.4108	0.3963	0.3865	0.3204	0.3059	0.3139	0.3222	0.3286
6	0.3945	0.3697	0.3455	0.3281	0.3159	0.4117	0.3652	0.3709	0.3819	0.3916
7	0.3316	0.3171	0.2920	0.2718	0.2566	0.5237	0.4244	0.4247	0.4387	0.4532
8	0.2753	0.2758	0.2513	0.2284	0.2100	0.6652	0.4829	0.4736	0.4904	0.5110
9	0.2257	0.2460	0.2242	0.1996	0.1781	0.8491	0.5399	0.5148	0.5333	0.5610
10	0.1830	0.2282	0.2119	0.1871	0.1632	1.0965	0.5936	0.5448	0.5624	0.5971

Table 2: Probability of Server Being Idle and Mean Queue Size
($\beta = 15, \mu = 15, \gamma = 15, s = 0.9, K = 4$)

λ	Probability of server being idle					Mean Queue Size				
	α =5	α =6	α =7	α =8	α =9	α =5	α =6	α =7	α =8	α =9
1	0.7294	0.7318	0.7341	0.7362	0.7382	0.0790	0.0801	0.0812	0.0821	0.0830
2	0.5888	0.5900	0.5910	0.5920	0.5929	0.1391	0.1486	0.1559	0.1617	0.1665
3	0.4636	0.4638	0.4640	0.4642	0.4643	0.1910	0.2139	0.2307	0.2436	0.2539
4	0.3537	0.3533	0.3529	0.3525	0.3522	0.2373	0.2775	0.3063	0.3282	0.3453
5	0.2592	0.2584	0.2577	0.2570	0.2564	0.2797	0.3402	0.3831	0.4152	0.4401
6	0.1805	0.1795	0.1786	0.1778	0.1771	0.3195	0.4023	0.4606	0.5037	0.5368
7	0.1180	0.1171	0.1162	0.1154	0.1146	0.3571	0.4633	0.5374	0.5918	0.6333
8	0.0725	0.0717	0.0709	0.0702	0.0695	0.3921	0.5215	0.6112	0.6767	0.7262
9	0.0446	0.0440	0.0434	0.0429	0.0424	0.4225	0.5740	0.6783	0.7539	0.8108
10	0.0353	0.0349	0.0346	0.0343	0.0341	0.4457	0.6162	0.7332	0.8175	0.8806

Table 3: Probability of Server Being Idle and Mean Queue Size
($\alpha = 10, \beta = 15, \gamma = 15, s = 0.9, K = 4$)

λ	Probability of server being idle					Mean Queue Size				
	$\mu = 15$	$\mu = 16$	$\mu = 17$	$\mu = 18$	$\mu = 19$	$\mu = 15$	$\mu = 16$	$\mu = 17$	$\mu = 18$	$\mu = 19$
1	0.7400	0.7448	0.7493	0.7534	0.7572	0.0839	0.0822	0.0806	0.0791	0.0777
2	0.5938	0.6040	0.6135	0.6224	0.6306	0.1705	0.1657	0.1611	0.1569	0.1529
3	0.4645	0.4790	0.4925	0.5051	0.5169	0.2624	0.2534	0.2451	0.2373	0.2300
4	0.3519	0.0697	0.3862	0.4017	0.4162	0.3592	0.3452	0.3323	0.3203	0.3091
5	0.2558	0.2759	0.2948	0.3125	0.3291	0.4599	0.4403	0.4221	0.4053	0.3895
6	0.1764	0.1981	0.2185	0.2377	0.2558	0.5630	0.5371	0.5131	0.4909	0.4702
7	0.1139	0.1366	0.1579	0.1780	0.1970	0.6659	0.6332	0.6030	0.5750	0.5491
8	0.0689	0.0919	0.1136	0.1340	0.1535	0.7648	0.7252	0.6886	0.6548	0.6235
9	0.0420	0.0647	0.0862	0.1066	0.1260	0.8548	0.8084	0.7656	0.7261	0.6895
10	0.0338	0.0558	0.0767	0.0965	0.1154	0.9291	0.8763	0.8279	0.7832	0.7418

Table 4: Probability of Server Being Idle and Mean Queue Size
($\alpha = 10, \beta = 15, \mu = 15, s = 0.9, K = 4$)

λ	Probability of server being idle					Mean Queue Size				
	$\gamma = 16$	$\gamma = 17$	$\gamma = 18$	$\gamma = 19$	$\gamma = 20$	$\gamma = 16$	$\gamma = 17$	$\gamma = 18$	$\gamma = 19$	$\gamma = 20$
1	0.7438	0.7471	0.7501	0.7528	0.7552	0.0801	0.0768	0.0739	0.0713	0.0690
2	0.6004	0.6062	0.6115	0.6162	0.6205	0.1625	0.1554	0.1492	0.1437	0.1387
3	0.4730	0.4806	0.4874	0.4936	0.4992	0.2496	0.2384	0.2285	0.2198	0.2119
4	0.3613	0.3699	0.3777	0.3848	0.3912	0.3414	0.3258	0.3120	0.2998	0.2889
5	0.2653	0.2741	0.2821	0.2895	0.2963	0.4371	0.4170	0.3993	0.3835	0.3694
6	0.1850	0.1932	0.2008	0.2080	0.2146	0.5355	0.5112	0.4896	0.4703	0.4530
7	0.1207	0.1275	0.1340	0.1403	0.1463	0.6345	0.6065	0.5814	0.5589	0.5387
8	0.0728	0.0773	0.0820	0.0868	0.0916	0.7310	0.7004	0.6727	0.6475	0.6247
9	0.0418	0.0431	0.0452	0.0479	0.0509	0.8209	0.7893	0.7602	0.7333	0.7087
10	0.0284	0.0254	0.0241	0.0240	0.0247	0.8984	0.8654	0.8396	0.8126	0.7872

Table 5: Probability of Server Being Idle and Mean Queue Size
($\alpha = 10, \gamma = 15, \mu = 15, s = 0.9, K = 4$)

λ	Probability of server being idle					Mean Queue Size				
	$\beta = 14$	$\beta = 15$	$\beta = 16$	$\beta = 17$	$\beta = 18$	$\beta = 14$	$\beta = 15$	$\beta = 16$	$\beta = 17$	$\beta = 18$
1	0.7341	0.7400	0.7452	0.7497	0.7538	0.0849	0.0839	0.0830	0.0822	0.0816
2	0.5885	0.5938	0.5984	0.6024	0.6061	0.1723	0.1705	0.1691	0.1678	0.1667
3	0.4600	0.4645	0.4685	0.4720	0.4752	0.2649	0.2624	0.2604	0.2588	0.2574
4	0.3484	0.3519	0.3550	0.3579	0.3605	0.3621	0.3592	0.3568	0.3549	0.3533
5	0.2537	0.2558	0.2578	0.2597	0.2615	0.4631	0.4599	0.4574	0.4554	0.4538
6	0.1763	0.1764	0.1768	0.1774	0.1781	0.5659	0.5630	0.5607	0.5590	0.5576
7	0.1166	0.1139	0.1121	0.1110	0.1102	0.6676	0.6659	0.6645	0.6635	0.6628
8	0.0752	0.0689	0.0642	0.0607	0.0581	0.7638	0.7648	0.7656	0.7661	0.7667
9	0.0530	0.0420	0.0336	0.0270	0.0219	0.8487	0.8548	0.8593	0.8628	0.8656
10	0.0509	0.0338	0.0206	0.0103	0.0021	0.9143	0.9291	0.9399	0.9481	0.9546

From Table 1, it is observed that the probability of server being idle decreases as the arrival rate increases from 1 to 10 for different values of K=2, 3, 4, 5, 6. From Table 2, it is observed that the probability of server being idle decreases as the arrival rate increases from 1 to 10 for different values of $\alpha = 5, 6, 7, 8, 9$. From Table 3, it is observed that the probability of server being idle decreases as the arrival rate increases from 1 to 10 for different values of $\mu = 15, 16, 17, 18, 19$. From Table 4, it is observed that the probability of server being idle decreases as the arrival rate increases from 1 to 10 for different values of $\mu = 16, 17, 18, 19, 20$. From Table 9, it is observed that the probability of server being idle decreases as the arrival rate increases from 1 to 10 for different values of $\mu = 14, 15, 16, 17, 18$.

6 Conclusion

In this article, a single server batch arrival, batch service queue with compulsory vacation and with unreliable server has been completely analyzed. We have derived some particular models by assuming particular values to the parameters. Also the numerical illustrations of the particular model considered are given in the form of tables. The model can be extended by taking the break down period as generally distributed.

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